Damage Detection on Mesosurfaces using Distributed Sensor Network and Spectral Diffusion Maps

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Abstract. In this work, we develop a data-driven method for the diagnosis of damage in mesoscale mechanical structures using an array of distributed sensor networks. The proposed approach relies on comparing intrinsic geometries of data sets corresponding to the undamage and damage states of the system. We use spectral diffusion map approach for identifying the intrinsic geometry of the data set. In particular, time series data from distributed sensors is used for the construction of diffusion maps. The low dimensional embedding of the data set corresponding to different damage levels is obtained using singular value decomposition of the diffusion map. We construct appropriate metrics in the diffusion space to compare the different data sets corresponding to different damage cases. The developed algorithm is applied for damage diagnosis of wind turbine blades. Towards this goal, we developed a detailed finite element-based model of CX-100 blade in ANSYS using shell elements. Typical damage, such as crack or delamination, will lead to a loss of stiffness, is modeled by altering the stiffness of the laminate layer. One of the main challenges in the development of health monitoring algorithms is the ability to use sensor data with relatively small signal-to-noise ratio. Our developed diffusion map-based algorithm is shown to be robust to the presence of sensor noise. The proposed diffusion map-based algorithm is advantageous by enabling the comparison of data from numerous sensors of similar or different types of data through data fusion, hereby making it attractive to exploit the distributed nature of sensor arrays. This distributed nature is further exploited for the purpose of damage localization. We perform extensive numerical simulations to demonstrate that the proposed method can successfully determine the extent of damage on the wind turbine blade and also localize the damage. We also present preliminary results for the application of the developed algorithm on the experimental data. These preliminary results obtained using experimental data are promising and is a topic of our ongoing investigation.

Keywords: sensor network, structural health monitoring, diffusion map, condition assessment, damage detection, diagnosis.
1. Introduction

Structural health monitoring (SHM) of large-scale systems, or mesosystems, including energy structures (e.g., wind turbine, dam), transportation infrastructures (e.g., bridge, pavement), and mechanical systems (e.g., aircraft, ship) is a difficult task due to the large geometries under inspection. Nevertheless, SHM at the mesoscale may have strong economic benefits. It has the potential to enable condition-based maintenance, instead of traditional time-based or breakdown-based strategies that are far less effective in terms of prolonging structural life. In particular, economic benefits for wind turbine blades are well understood [1, 2, 3].

To cope with the mesoscale challenge, off-the-shelf sensing strategies need to be adapted to provide large-area sensing capabilities [4]. A solution is to deploy distributed sensor networks (DSNs), which include wireless [5, 6] and multivariate [7, 8, 9, 10] networks, as well as dense arrays of sensors [11, 12, 13, 14, 15] that mimic biological skins, where changes in a local state can be monitored over a global area. The application of DSNs for SHM purposes typically leads to a significant quantity of data that needs to be processed strategically in order to obtain features related to structural condition. This is generally done using physics-driven or data-driven methods. While physics-driven methods typically lead to more accurate prognosis, they often rely on complex models that require long computation time. Conversely, data-driven methods can be operated in real-time and can be used to quickly detect a change in a condition, but yield results that may be difficulty to relate to structural behaviors [16].

The objective of this paper is to develop a condition assessment method for mesoscale systems that leverages the utilization of DSNs and fuses sensor data into a condition index. We selected a data-driven method due to the fast computational time that may lead to real-time applications. The method is based on spectral diffusion maps. Diffusion maps belong to unsupervised learning algorithms dealing with a spectral analysis of non-linear data and requires no prior knowledge regarding the appearance of damage, and no use is made of training data. With this method, the intrinsic geometries of the data sets obtained from DSNs are compared to identify potential changes in the system states, which would indicate damage. The intrinsic geometry of the data set is obtained using the multiscale diffusion map approach developed in [17]. The diffusion-map method provides an embedding of the time-series data set in the diffusion space to identify important lower dimensional dynamic features of data. We construct appropriate metrics in the diffusion space to compare the embedded data under normal and abnormal operating conditions.

In the following we provide a brief overview of literature on comparison of intrinsic geometry of data sets. In [18], a multivariate attractor-based approach is used to detect the presence and magnitude of damage in structures through the investigation of the response’s phase-space constructed by a time delayed embedding. A metric is introduced to quantify the damage-sensitive feature by comparing with the attractor of the undamaged structural response. Ref. [19] used the attractor constructed from the
undamaged state to predict structural response, and identified damage as a change in the prediction error. An approach [20] applied the theory to large nonlinear systems by dividing the system into a set of subsystems, and time series responses of each subsystem analyzed to identify damage. The authors in [21] proposed to analyze nonlinear time series using a multivariate autoregressive (MAR) approach in order to detect damage under varying operational and environmental conditions. Ref. [22] used a combined state-space embedding strategy and singular value decomposition to detect structural damage. In [23], a diffusion map-based approach was used for detection of anomaly in dynamic systems [23]. Ref. [24] proposed a variation of diffusion maps termed discriminant diffusion maps analysis (DDMA) machine condition monitoring and fault diagnosis. The algorithm for diffusion map-based data comparison used in this paper was first presented in [25]. In [26], the theoretical basis for the construction and comparison of diffusion maps for family of data set changing with respect to change in system parameters is provided.

The diffusion map-based approach presented in this paper combines ideas from a variety of methods currently adopted for data-driven schemes for health monitoring such as spectral graph theory, Kernel methods, and machine learning. One of the important advantages of the proposed diffusion map-based approach is that it can be used for sensor data fusion. The presented algorithms exploit the distributed nature of sensor data in the form of sensor fusion. Comparison based on fused data from multiple sensors has the advantage that it is relatively robust to sensor noise thereby making it attractive in dealing with sensors with small signal-to-noise (SNR) ratio. Furthermore, DSN also provides an opportunity to localize the damage on the structure. Most of damage localization methods are applicable at a localized area, but not economically feasible in a large-scale structure [27]. Damage localization in structures allows for considerable reduction of expenses related to their operation as well as increase in safety and longer lifespan. We show that our proposed approach successfully makes use of DSN to localize the damage on a wind turbine blade. Results presented in this paper are an extended version of the paper published in the Proceedings of American Control Conference [28]. The main contributions of this paper are as follows. A nonlinear dimensionality-reduction framework using diffusion maps for structural condition assessment based on the intrinsic geometries of the data is proposed. This approach provides a low dimensional representation for a given set of heterogenous sensors which combines all the sensor information and the metric constructed is used to measure the connectivity in data points and achieves relatively robustly with respect to sensor noise. We also demonstrate that the proposed approach is well suited for identifying and locating the damage in the structures using DSN.

The organization of the paper is as follows. Section 2 reviews the underlying theory of diffusion maps. Section 3 describes the application of diffusion maps to different data sets and the sensor fusion strategy. Section 4 presents the numerical simulation models. Section 5 presents and discusses simulation results. Section 6 presents preliminary experimental validation results. Section 7 concludes the paper.
2. Diffusion Map and Algorithm

In this section, we provide the background on diffusion maps, which constitute the basis of our approach. The theory and the discussion presented in this section closely follows from [17, 29, 25]. The construction of diffusion maps starts with the construction of a kernel function, \( k(x, y) \), on set of data points \( \Gamma \), where \( x \) and \( y \) are data points belongs to the space \( \Gamma \). The \( \Gamma \) here refers to the underlying high dimensional Euclidean space where the data points lies. The goal is to extract the lower dimensional intrinsic manifold which best describe the data set in the high dimensional Euclidean space \( \Gamma \) and this goal is accomplished through the process of diffusion map. The kernel function \( k(x, y) \) is constructed satisfying the following properties:

- \( k \) is symmetric, i.e., \( k(x, y) = k(y, x) \)
- \( k \) is positivity preserving i.e., \( k(x, y) \geq 0 \) for all \( x, y \) on \( \Gamma \).
- \( k \) is positive semidefinite for all real valued bounded function \( f \) defined on the data set \( \Gamma \),

\[
\int_{\Gamma} \int_{\Gamma} k(x, y) f(x) f(y) d\mu(x)d\mu(y) \geq 0,
\]

where \( \mu \) is a probability measure describing the distribution of points in \( \Gamma \).

The kernel function \( k(x, y) \) is constructed based on local connectivity of data points and hence capture the local geometry of data set. The local geometries are integrated through the process of diffusion map to provide information on the global geometry i.e., the shape of the intrinsic lower dimensional manifold on which the data points reside. Several choices for the kernel \( k \) are possible, all leading to different analyzes of data. In this paper, we use the Gaussian or exponential form for the kernel function. The kernel function is used for the construction of the global geometry of data. The first step towards the construction of the diffusion map is to normalize the kernel function \( k(x, y) \) as follows [30]. For all \( x \in \Gamma \)

\[
\text{let } v^2(x) = \int_{\Gamma} k(x, y) d\mu(y),
\]

and set

\[
\tilde{a}(x, y) = \frac{k(x, y)}{v^2(x)}.
\]

It follows from the construction that \( \int_{\Gamma} \tilde{a}(x, y) d\mu(y) = 1 \). To \( \tilde{a} \) we can associate a linear operator on the data set \( \Gamma \) as follows:

\[
(\tilde{A} f)(x) = \int_{\Gamma} \tilde{a}(x, y) f(y) d\mu(y).
\]

Since we are interested in the spectral properties of the operator, it is preferable to work with a symmetric conjugate of \( \tilde{A} \). We conjugate \( \tilde{a} \) by \( v \) in order to obtain a symmetric form and we consider

\[
a(x, y) = \frac{k(x, y)}{v(x)v(y)},
\]
and the operator
\[ (Af)(x) = \int_{\Gamma} a(x, y)f(y)d\mu(y). \] (3)

The operator \( A \) is termed diffusion operator. Under very general hypotheses, the operator \( A \) is compact and self-adjoint. Thus, by spectral theory, we have

\[ a(x, y) = \sum_{j \geq 0} \lambda_j \varphi_j(x)\varphi_j(y), \quad A\varphi_j(x) = \lambda_j \varphi_j(x). \] (4)

where \( \varphi_j(x) \) are eigenfunctions of \( A \) corresponding to eigenvalue \( \lambda_j \). Let \( a^m(x, y) \) be the kernel of \( A^m \), then at the level of data points the kernel \( a^m(x, y) \) has a probabilistic interpretation as a Markov chain with transition matrix \( a \) to reach \( y \) from \( x \) in \( m \) steps. Now define a mapping \( \Phi : \Gamma \to \ell^2(N) \) as

\[ \Phi(x) = (\varphi_0(x), \varphi_1(x), ..., \varphi_p(x), ...), \]

mapping the data point \( x \in \Gamma \) into the Euclidean space \( (\ell^2(N)) \), which we will call the diffusion space. Each eigenfunction can be interpreted as a coordinate on the set. The diffusion distance in the original space \( \Gamma \) can now be defined using the mapping \( \Phi \). In particular, diffusion distance between two points \( x, y \in \Gamma \) after \( m \) time steps is defined as follows

\[ D^2_m(x, y) = \sum_{j \geq 0} \lambda_j^m (\varphi_j(x) - \varphi_j(y))^2. \] (5)

Note that the diffusion distance between two points in the original space \( \Gamma \) is simply the Euclidean distance in the diffusion space. The diffusion distance measures the local connectivity between the points in the underlying data set. Its value depends on the number of connecting paths between data points.

The diffusion map is used to map coordinates between data and the diffusion space, and can be exploited for dimensionality reduction. Dimensionality reduction can be conducted from the embedding generated by the eigenfunctions. For a given accuracy \( \delta \) we retain only the eigenvalues \( \lambda_0, ..., \lambda_{p-1} \) that, when raised to the power \( m \), exceed a certain threshold (related to delta), and we use the corresponding eigenfunctions \( \varphi_0, \varphi_1, ..., \varphi_{p-1} \) to embed the data points in \( \mathbb{R}^p \).

3. Comparison of data sets and sensor fusion

The underlying idea behind the comparison of data sets using the diffusion map approach is adopted from [25]. For the simplicity of presentation, we will explain the comparison procedure between two data sets \( X \) and \( Y \). The procedure for comparison involving multiple data sets is straightforward. Let \( X = \{x_1, x_2, ..., x_N\} \) and \( Y = \{y_1, y_2, ..., y_N\} \) be the two data sets obtained in the form of time series from an experiment or model simulation. We are assuming that the two data sets are of the same size, as this is our case of interest. However, the approach can be extended to the case when data sets are of different sizes [26]. Using time-delayed coordinates, we embed the time series
data in $R^n$, where $n$ is sufficiently large. Now we have $N-n$ data points denoted by $\tilde{X} := \{\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_{N-n}\}$, $\tilde{Y} := \{\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_{N-n}\}$, where $\tilde{x}_k = (x_k, x_{k+1}, \ldots, x_{k+n-1})$ and $\tilde{y}_k = (y_k, y_{k+1}, \ldots, y_{k+n-1})$. We denote the union of these two data sets by $Z = \{\tilde{X}, \tilde{Y}\}$. In this paper, we use the following Gaussian kernel function,

$$k(z_k, z_j) = e^{-\frac{||z_k - z_j||^2}{\epsilon}},$$

(6)

The parameter $\epsilon$ is important in the computation of the Gaussian kernel. It is highly data dependent and specifies the size of the neighborhoods defining the local geometry of the data. The smaller the parameter $\epsilon$, the faster the exponential decreases and hence the weight function in (6) becomes numerically insignificant as we move away from the center. It is easy to verify that the Gaussian kernel satisfies all the properties of the kernel specified in the previous section.

From this kernel, we construct the diffusion operator or the diffusion matrix as follows. Assuming the transition probability between points $z_k, z_j$ is proportional to $k(z_k, z_j)$, we can construct the Markov matrix as follows

$$M(k, j) = \frac{k(z_k, z_j)}{p(z_j)},$$

(7)

where $p(z_k)$ is the normalization constant given by

$$p(z_j) = \sum_k k(z_k, z_j)$$

(8)

Finally, the singular value decomposition is applied to $M$, yielding eigenvalues $\lambda$, which are sorted in descending order, and the corresponding eigenvectors $\varphi$. The eigenvalues of $M$ lie in the range 0 to 1 due to normalization. Let $\{\varphi_1, \varphi_2, \ldots, \varphi_{2(N-n)}\}$ be the eigenvectors of the diffusion matrix and $\{\lambda_1, \lambda_2, \ldots, \lambda_{(N-n)}\}$ be the corresponding eigenvalues. Retaining only the first $p$ eigenvectors and eigenvalues ($p = \max\{l \in \mathbb{N} \text{ such that } |\lambda_l| > \delta |\lambda_1|\}, \delta > 0$ [31]) we can embed the data set $Z$ in a $p$-dimensional Euclidean diffusion space, where $\{\varphi_1, \ldots, \varphi_p\}$ are the coordinates of the data points in the Euclidean space. Note that typically $p \ll n$ and hence we obtain the dimensionality reduction of the original data set. For some index $j$, the first $N-n$ elements of the eigenvector $\varphi_j$ are the $j$-th coordinate in the diffusion space of the $N-n$ data points in $X$, while the remaining $N-n$ elements are the $j$-th coordinate in the diffusion space of the data set $Y$. Denote the eigenvector on data set $X$ by $\varphi^X$ and data set $Y$ by $\varphi^Y$:

$$\varphi := \begin{bmatrix} \varphi^X \\ \varphi^Y \end{bmatrix}.$$  

(9)

Note that the $k$-th elements of the $j$-th eigenvectors are given, respectively, by

$$\varphi^X_{kj} := \varphi^X_j(\tilde{x}_k), \quad \varphi^Y_{kj} := \varphi^Y_j(\tilde{y}_k).$$

(10)

We can use various metrics for the comparison of data sets in diffusion space using the above eigenvectors. We define

$$\phi^X_k = \left(\sum_{j=1}^{p} \lambda_j (\varphi^X_{kj})^2\right)^{\frac{1}{2}}, \quad \phi^Y_k = \left(\sum_{j=1}^{p} \lambda_j (\varphi^Y_{kj})^2\right)^{\frac{1}{2}}$$

(11)

and propose following metric for the comparison of data sets.
(i) **Weighted average diffusion distance**

\[
D_{\text{avg}} = \left[ \frac{1}{N - n} \sum_{k=1}^{N-n} \phi_k^X \right] - \left[ \frac{1}{M - n} \sum_{k=1}^{M-n} \phi_k^Y \right]
\]  

(ii) **Pointwise diffusion distance**

\[
D_p = \frac{1}{N} \sum_{k=1}^{N} \frac{|\phi_k^X - \phi_k^Y|}{\phi_k^X}
\]  

This metric is sensitive to the ordering of the data set. Other metrics can also be constructed depending upon application [32]. For our proposed application of damage diagnosis of wind turbine blades, we employ the pointwise diffusion distance for data comparison in the diffusion space. This metric measures the diffusion distance between the data points in the original space and provides robust information on the geometry of the data set. The pointwise distance metric gives us satisfactory results. The proposed approach for the comparison of two data sets can be extended to multiple data sets in a straightforward manner [25]. For our proposed application, the different data sets will correspond to the different damage levels of a wind turbine blade. While the above procedure helps us compare different data sets corresponding to different damage levels, the procedure can be extended for comparison of data sets from multiple sensors. This can be accomplished using sensor fusion. We consider the case where the wind turbine blade is equipped with an array of distributed sensors. The goal is to fuse data from multiple sensors for damage diagnosis and also for damage localization.

### 3.1. Multiple sensor fusion

The procedure for sensor fusion in reconstructing the state of dynamical systems using diffusion maps is described in [33]. The strategy is to construct hierarchies of diffusion maps for a system consisting of heterogeneous sensors, where each sensor can be parameterized and normalized in its intrinsic diffusion coordinates, and a new graph is generated by combining all of the relevant diffusion coordinates from all the sensors. The algorithm for the multiple sensor fusion as it applies to our problem of damage detection is given below. The algorithm closely follows one used in [23] except for the comparison metric that is defined above in Eq. (13). For simplicity and conciseness, we will only consider the case of data fusion from three sensors.

**Comparison of different damage data sets using multiple sensors**

(i) Let \(X_i = \{x_1^i, x_2^i, ..., x_N^i\}, Y_i = \{y_1^i, y_2^i, ..., y_N^i\}, \) and \(Z_i = \{z_1^i, z_2^i, ..., z_N^i\}\) be the data sets from three sensors. The index \(i = 0, 1, 2, 3...\) is the index for damage, with 0 is for undamaged case and \(N\) is the length of each data set. Using time delayed coordinates, we embed \(X_i\) for each \(i\) in \(\mathbb{R}^n\) where \(n\) is sufficiently large.

(ii) We have \(N - n\) data points for individual time series \(\bar{X}_i := \{\bar{x}_1^i, \bar{x}_2^i, ..., \bar{x}_N^i\}\) where \(\bar{x}_k^i = (x_k^i, x_{k+1}^i, ..., x_{k+n-1}^i)\) We denote the union of these data sets \(\bar{X}_0, \bar{X}_1, \ldots\) as \(\bar{X} = \{\bar{X}_0, \bar{X}_1, \ldots\}\)
(iii) We apply the procedure outlined above to other sensors $Y, Z$, and we get $\hat{Y}$ and $\hat{Z}$.
(iv) We apply the diffusion map to data set $\hat{X}$. The embedding coordinates of $\hat{X}$ are scaled and are denoted by $\Psi_1$ as $\Psi_1(x) = (\lambda_1 \psi_1(x), \lambda_2 \psi_2(x), \lambda_3 \psi_3(x), \ldots)$.
(v) We repeat the above procedure for all of the different data sets $\hat{Y}$ and $\hat{Z}$, and the scaled embedding coordinates for $\hat{Y}$ and $\hat{Z}$ is given by $\Psi_2$ and $\Psi_3$.
(vi) The scaled diffusion coordinates are combined into a matrix form given by $W = \{\Psi_1, \Psi_2, \Psi_3\}$. The diffusion map is applied again on this matrix $W$.
(vii) We retain only the first $p$ eigenvectors ($p \ll n$) of the diffusion matrix and $\{\lambda_1, \lambda_1, \ldots, \lambda_p\}$ corresponding eigenvalues, so that we can embed the data set $W$ in a $p$-dimensional Euclidean diffusion space.
(viii) The resulting eigenvectors can be decomposed in the form of damage indices as $\hat{\phi} = [\hat{\phi}^0; \hat{\phi}^1; \hat{\phi}^2; \ldots]$
(ix) The pointwise diffusion distance is applied on these sets of eigenvectors in order to capture the varying degrees of damage in the system.

A schematic of the sensor fusion approach is shown in Figure 1 using $n$ sensors.

![Figure 1: Sensor fusion using $n$ sensors](image)

4. Methodology

In this section, we present the numerical model used for the numerical analysis of the proposed method. The model consists of a wind turbine blade equipped with a DSN and subjected to various wind loads, described in what follows.

4.1. Wind turbine blade model

The wind turbine blade is modeled after the 9 m CX-100 carbon fiber blade described in [34]. This particular blade has been widely studied [35, 36, 37]. A simplified finite
element model was generated in ANSYS using shell elements. It consists of a tapered cantilever plate of 9 m length, 1.03 m largest width, and 0.035 m thickness, as shown in Fig. 2. The blade is a composite assembled from 3 different layers constituted with 2 different materials and 3 different orientations, as listed in Table 1.

![Blade Dimensions](image)

**Figure 2:** Wind turbine blade dimensions (mm) (a) top view; and (b) cross section.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material (orientation)</th>
<th>$E_x$ (GPa)</th>
<th>$E_y$ (GPa)</th>
<th>$G_{xy}$ (GPa)</th>
<th>density (kg/m$^3$)</th>
<th>thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Carbon-fiberglass fabric (+45°)</td>
<td>84.10</td>
<td>8.76</td>
<td>4.38</td>
<td>3469</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>C520 fiberglass (0°)</td>
<td>37.30</td>
<td>7.60</td>
<td>6.89</td>
<td>1874</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>Carbon-fiberglass fabric (−45°)</td>
<td>84.10</td>
<td>8.76</td>
<td>4.38</td>
<td>3469</td>
<td>13</td>
</tr>
</tbody>
</table>

The blade was modeled to match the first flatwise and edgewise frequencies of the experimental values reported in Ref. [37]. The model and experimental values are compared in Table 2. The first frequencies of the model agree with the experimental values.

**Table 2:** Comparison of frequencies

<table>
<thead>
<tr>
<th>direction</th>
<th>model (Hz)</th>
<th>experimental (Hz) [37]</th>
<th>difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>flapwise</td>
<td>4.16</td>
<td>4.56</td>
<td>−8.8</td>
</tr>
<tr>
<td>edgewise</td>
<td>8.02</td>
<td>7.49</td>
<td>+7.1</td>
</tr>
</tbody>
</table>

4.1.1. **Damage Cases** Five different damage locations and severities are considered in the simulations. They are schematized in Fig. 3, in which the red-dashed regions represent the damaged element. Damage locations 1 to 4 (Fig. 3(a)-(d)) vary from the root (Fig. 3(a)) to the free end (Fig. 3(d)) of the blade, while damage location 5 (Fig. 3(e))) is a combination of damage locations 1 and 3. The blue dots represent the location of 9 virtual strain gauges constituting the DSN. The simulated measured data is surface strain in the form of a time history sampled at 10 Hz. They are equally spaced at 1 m and located in the middle of blade. Fig. 3(f) shows the cartesian coordinates of the nine virtual strain gauges.
Figure 3: Damage locations under study: (a) location 1; (b) location 2; (c) location 3; (d) location 4; (e) location 5; and (f) sensors location.

The simulated damage is a loss of stiffness arising from delamination, a damage mode commonly studied in wind turbine blade literature [3]. It is modeled as a change in the stiffness of laminate layer 2. Five different damage severities are considered under damage location 1 (Fig. 3(a)), which correspond to changes in the first natural frequencies of 1%, 2%, 5%, 10% and 15% (35.5%, 54.8%, 80.6%, 92.3% and 96.7% stiffness loss in damaged elements in the strong bending direction). The damage localization study compares all locations under constant damage corresponding to a 10% change in the first natural frequency.

4.2. Wind Load model

The natural variability of the wind loading on the blade is generated using the procedure described in [38]. The wind speed $W_s$ applied to the wind turbine blade is constituted from four components:

$$W_s = W_a + W_r + W_g + W_t,$$ (14)
where $W_a$ is the average speed, $W_r$ is the ramp component, $W_g$ is the gust component, and $W_t$ is the turbulence. The ramp component $W_r$ is taken as

$$W_r = \begin{cases} 
0 & \text{if } t < T_{sr} \\
w_{\text{ramp}} & \text{if } T_{sr} < t < T_{er} \\
0 & \text{if } t > T_{er}
\end{cases}$$

(15)

where $w_{\text{ramp}} = A_{\text{ramp}} \frac{(t-T_{sr})}{(T_{er}-T_{sr})}$ with $A_{\text{ramp}}$ being the amplitude of wind speed ramp, $T_{sr}$ and $T_{er}$ are the starting and end time of wind speed ramp, respectively. The wind gust $W_g$ is taken as

$$W_g = \begin{cases} 
0 & \text{if } t < T_{sg} \\
w_{\text{gust}} & \text{if } T_{sg} < t < T_{eg} \\
0 & \text{if } t > T_{eg}
\end{cases}$$

(16)

where, $w_{\text{gust}} = A_{\text{gust}} \left(1 - \cos \left(2\pi \left(\frac{t-T_{sg}}{T_{eg}-T_{sg}}\right)\right)\right)$ with $A_{\text{gust}}$ being the amplitude of wind gust, $T_{sg}$ and $T_{eg}$ are the starting and end time of wind gust, respectively. $W_t$ is modeled as a one-dimensional random process and is characterized by the following power spectral density function $P(f)$ for a given frequency $f$ [38]

$$P(f) = l \cdot W_a \left(\ln \left(\frac{h}{z_0}\right)^2\right)^{-1} \left(1 + 1.5 \frac{f}{W_a} \right)^{-5/3},$$

where $l$ is the turbulence length scale, $h$ is the height at which the wind speed is applied, and $z_0$ is the roughness length. The wind pressure acting on the blade is directly obtained from $W_s$ using [39]

$$W_p = 0.5\rho W_s^2,$$

(17)

where $\rho$ is the air density. The variability in wind speed at different heights across the blade is taken into account using the power law [40]. The resulting wind pressure obtained (17) is applied onto the top surface of the wind turbine blade model. Table 4.2 lists the values of the selected parameters for the generation of different wind load realization. In order to take into account the uncertainty in wind speed, each damage case is simulated under three different wind pressure realization using the parameters listed in Table 4.2. A total of 30 different realizations are considered in this paper. Fig. 4 shows three different realizations of wind pressure.
5. Simulation Results

In this section, we study the performance of the diffusion map algorithm at detecting different damage levels and locations.

5.1. Different damage levels

Figure 5 shows the study of the eigenvalues of the diffusion map obtain from sensor 1 only, for the undamaged blade subjected to a wind load realization. Other than the first eigenvalue at one, there are three dominating eigenvalues (choosing $\delta = 0.01$, we have $p = 3$, refer to paragraph below equation 8). Thus, the data set can be approximated using the three dominant eigenvectors of the diffusion map. The eigenvector plot corresponding to first three dominant eigenvalues for all the damage cases is shown in Fig. 5(b).

The exercise is repeated for sensor 2, which is the closest to the damage. Similarly to sensor 1, the study of the eigenvalues (Fig. 6(a)) shows three dominant eigenvalues. A plot of the eigenvectors of sensor 2 data set is shown in Fig. 6(b). A comparison between the eigenvector plots for sensor 1 (Fig. 5(b)) and sensor 2 (Fig. 6(b)) shows a more apparent change in the magnitude of the eigenvectors as the damage increases. This is largely attributed to the larger change in strain readings from sensor 2, as it is closer to the damage.

Figure 8 is a plot of the pointwise diffusion distances $D_p$ for all the nine sensors as a function of different damage cases, where 0% corresponds to the undamaged case. As noted previously, three different wind load realizations were simulated for each damage case. The average data from three different wind realizations for each damage case is used for calculating the pointwise diffusion distance using the proposed approach. The results for each sensor shows an increasing $D_p$ for an increasing damage level. The pointwise diffusion distance $D_p$ can therefore be utilized to detect and evaluate multiple damage cases and to monitor the structural health of the blade.

Table 3: Model parameters for wind load generation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$T_{sr} = T_{sg}$</td>
<td>50 s</td>
</tr>
<tr>
<td>$T_{er}$</td>
<td>150 s</td>
</tr>
<tr>
<td>$T_{eg}$</td>
<td>200 s</td>
</tr>
<tr>
<td>$h$</td>
<td>70 m</td>
</tr>
<tr>
<td>$A_{ramp}$</td>
<td>4 m/s</td>
</tr>
<tr>
<td>$A_{gust}$</td>
<td>-3 m/s</td>
</tr>
<tr>
<td>$l$</td>
<td>600 m</td>
</tr>
<tr>
<td>$z$</td>
<td>0.01 m</td>
</tr>
<tr>
<td>$W_a$</td>
<td>11.5 m/s</td>
</tr>
</tbody>
</table>
the gravity of damage. Sensor 2 (Sen 2) exhibits a notably higher magnitude of $D_p$ compared with other sensors. This demonstrates that $D_p$ can also be utilized to localize damage.

The sensor fusion strategy described in Section 3 can be used to provide a direct measure of damage. In Fig. 9, the information from all the 9 sensors is fused. Results show an increasing pointwise diffusion distance with increasing damage level. A relationship between $D_{SF}$ and damage levels could be established to create a useful damage index, enabling damage prognosis.

Results discussed above demonstrate that the embedding of the map can be used for damage detection. Natural question arise if we can devise a simpler algorithm to accomplish what we have using the proposed diffusion map approach. In the following, we describe a relatively simple algorithm adopted from [41]. We show that
algorithm can detect damage, the algorithm is not robust and is sensitive to noise. The following algorithm use only output strain data for the detection of damage. The
algorithm consists of comparing the relative response between two sensors through events. Assuming that the response of the dynamic system is largely dominated by the first mode (not to confuse with dominating eigenvalues of the diffusion map), the relative strain \( \varepsilon_i/\varepsilon_j \) between two points \( i \) and \( j \) remains approximately constant. Thus, we can write a performance index \( J \):

\[
J = \left| \left( \sum_{k=1}^{K} \frac{s_{i,k}}{s_{j,k}} - \sum_{k=1}^{K^*} \frac{s_{i,k}^*}{s_{j,k}^*} \right) \left( \sum_{k=1}^{K^*} \frac{s_{i,k}^*}{s_{j,k}^*} \right)^{-1} \right|
\]

where \( s_{i,k} \) is the signal of the sensor \( i \) at time \( k \), which is compared with \( s_{j,k} \), the signal of sensor \( j \) at time \( k \), over the time series \( K \). The star represents data associated with the undamaged case. This performance index \( J \) represents the change in the relative response between two sensors. It is used as a metric to assess the performance of the proposed algorithm. Note that \( s_{j,k} \) might not be limited to one sensor in the case where the comparison is conducted between two neighbors. The study on damage cases is repeated using this algorithm, where sensor \( i \) is compared with neighbors \( i \pm 1 \) for \( i = 2, 3, ..., 8 \) and \( i + 1 \) for \( i = 1 \) and \( i - 1 \) for \( i = 9 \). Figure 10 plots the value of \( J \) per sensor for different damage cases. Results show that damage is between sensors 1 and 2 or sensors 2 and 3 (highest \( J \) values). Also, similarly to the diffusion map algorithm, the \( J \) index can be used to detect, localize, and evaluate the gravity of damage. The comparison of the diffusion map algorithm with this simple study of the relative response will be useful, later in this section, to demonstrate the superior robustness of the proposed method.

![Figure 10: Performance \( J \) for various sensors.](image)

5.2. Different damage locations

To further demonstrate the capacity of the diffusion map algorithm at localizing damage, the algorithm is simulated for the other damage locations discussed in Section 4. Figure 11 is a plot of the pointwise diffusion distance values obtained by comparing against
the undamaged cases, as a function of each sensor. The magnitude of $D_p$ corresponding to sensor 2 is larger compared to all the sensors for location 1, sensor 3 for location 2, sensor 4 for location 3, sensor 5 for location 4, and sensors 2 and 4 for location 5. These values correspond to the closest sensors for each damage case, showing that the proposed methodology performs well at localizing each damage. The drift of that occurs at the end of the blade for each damage location is attributed to differences in the strain magnitudes with respect to the undamaged case.

![Graph](image)

**Figure 11**: Pointwise distance for all sensors for damage localization.

### 5.3. Robustness

The robustness of the diffusion map algorithm with respect to noise is investigated. Data sets are generated with different levels of noise: 0.1%, 1%, 5%, and 10%. The noisy data $s_{\text{noise}}(t)$ is generated from the actual data set $s_{\text{actual}}(t)$:

$$s_{\text{noise}}(t) = s_{\text{actual}}(t) + \sigma_{\text{noise}} \xi(t)$$

where the noise variance is given by $\sigma_{\text{noise}}^2 = \frac{\sigma_{\text{signal}}^2}{\text{SNR}}$ and $\xi(t)$ is a normally distributed random variable. The value of SNR is varies to add different levels of noise to the actual data set. Figure 12 shows the diffusion distances at different damage levels under these levels of noise. At 0.1% and 1% noise levels, it is clear that sensor 2 has the largest value for $D_p$, enabling damage localization. However, beyond 5% noise, damage below 1% is difficult to localize. At 10% noise, damages under 1% are difficult to localize. Nevertheless, damages cases at and beyond 5% changes in the dominating frequency are clearly identified, even at a 10% noise level.

The study can be extended to the relative strain comparison algorithm discussed above. Figure 13 shows the evolution of performance index $J$ as noise is increased. It becomes rapidly difficult to detect and localize data using this technique, which further exhibits the advantageous robustness of the proposed method.
Figure 12: Pointwise distance for different noise levels (0.1%, 1%, 5%, 10%).
Figure 13: Performance $J$: (a) 0.1% percent; (b) 1% percent (c) 5% percent (d) 10% percent.

6. Preliminary experimental results

A preliminary study on the experimental validation of the proposed methodology for damage detection is performed using laboratory data presented in [42]. Briefly, the experiment utilizes a network of four resistive strain gauges (RSGs) with resolution of $1 \mu\epsilon$ (Vishay Micro-Measurements, CEA-06-500UW-120), identified as RSG1 to RSG4, where RSG1 is the sensor close to the root (fixity), and RSG4 is the sensor close to the free end. A cantilever steel beam is used as a test case with dimensions $26.5 \times 6 \times 0.5$ in$^3$ subjected to a dynamic load applied near the free end. The excitation consists of a displacement-controlled frequency sweep ranging from 0.25 Hz to 15 Hz at 0.35 in amplitude, applied using an MTS universal testing machine. Data from the RSGs are acquired using a Hewlett-Packard 3852 DAQ at 275 Hz. Figure 14 is a picture of the experimental setup. Note that the setup was simultaneously used to test the performance of other sensors (the four large black squares shown in the figure), which were unrelated to the algorithm presented in this paper. Damage was induced at the
root of the beam between the fixity and RSG1 by sequentially reducing the cross-section area by 1%, 2%, 4%, 5%, and 8%.

Figure 15(a) shows the pointwise diffusion distance for each sensor. The diffusion distance $D_p$ increases with the increase in the extent of the damage. This diffusion distance plot obtained using experimental data is consistent with the one obtained using the simulated data thereby validating the fact that the proposed algorithm works on the experimental data. However, no RSG sensors shows a clear difference with respect to other sensors except for RSG4 that shows a relatively lower $D_p$, as expected from the results of the numerical simulations. In this case, damage could only be localized in the vicinity of RSG1 to RSG3. This loss of resolution compared with the numerical results can be attributed to the low signal-to-noise ratio provoked by noise and the low level of strain applied on the specimen (two orders of magnitude smaller than the simulated strain in the numerical simulations). Damage localization using the proposed algorithm will require more careful investigation and will be the topic of our future publication. Figure 15(b) shows the point-wise diffusion distance for all sensors using data fusion. The metric $D_{SF}$ metric increases sequentially with respect to the level of damage, with a trend similar to the one obtained from the results of the numerical simulations.

Figure 14: Schematic of experimental setup.
7. Conclusion

In this work, we proposed a new approach enabling damage detection and localization on mesosystems. The approach consists of utilizing a sensor network combined with a spectral diffusion map-based method. With the diffusion maps, the intrinsic geometries between two data sets are compared, and a change in these geometries is an indicator of damage. The magnitude of such change can be used to compare the magnitude of damage, the first step towards prognosis. By comparing the diffusion distances at each sensor, it is also possible to localize damage. An algorithm for data fusion as been presented, which enables the combination of multiple data sets from a number of sensors, which may measure different states, for damage diagnosis.

The proposed method has been investigated via numerical simulations. These simulations were conducted on a realistic blade model subjected to different wind realizations. Different damage cases and localizations have been used to study the performance of the algorithm. Results showed that, without noise, the method was able to locate and detect damage as low as 0.1%. In the presence of noise, the method was able to locate and detect a 0.1% damage under a 1% noise level, and a 5% damage under a 10% noise level. Results were also compared with a simple comparison of relative responses between sensors, which failed at providing an acceptable damage detection and localization performance under noise. The data fusion algorithm was successful at providing an overall measure of damage.

This study demonstrated critical advantages of the proposed approach. First, the spectral diffusion map-based method can be combined with DSNs to locate and detect damage. Second, it can be used to fuse information from multiple sensors to provide a numerical value linked to a measure of damage gravity. Third, it is robust with respect to noise. It follow that the proposed approach has great potential for structural health
monitoring of mesosystems.

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