

Math 165 (Chris Kurth)
Spring 2008
Quiz 12

Show all work. Answers without work will not receive credit.

1. (6 points) Find $\frac{d}{dx} \log_{10}(x^3 + 9)$

Solution:

$$\begin{aligned} \frac{d}{dx} \log_{10}(x^3 + 9) &= \frac{d}{dx} \frac{\ln(x^3 + 9)}{\ln 10} \\ &= \frac{1}{\ln 10} \cdot \frac{3x^2}{x^3 + 9} \end{aligned}$$

2. (6 points) Find $\int 10^{5x-1} dx$

Solution: Let $u = 5x - 1$, so $du = 5dx$. Then

$$\begin{aligned} \int 10^{5x-1} dx &= \frac{1}{5} \int 10^u du \\ &= \frac{1}{5 \ln 10} 10^u + C \\ &= \frac{1}{5 \ln 10} 10^{5x-1} + C \end{aligned}$$

3. (8 points) The number of bacteria in a certain culture grows from 50 to 400 in 12 hours. If the rate of increase is proportional to the number of bacteria present, how many bacteria will be present after 16 hours?

Solution: Let the number of bacteria at time t be $A(t)$, and we know $A(t) = A_0 e^{kt}$. Then $50 = A(0) = A_0$ and $400 = A(12) = 50e^{12k}$. So $8 = e^{12k}$ and $\ln 8 = 12k$ and $k = \frac{\ln 8}{12}$. Thus:

$$\begin{aligned} A(16) &= 50e^{\frac{\ln 8}{12} \cdot 16} \\ &= 50e^{\frac{4}{3} \ln 8} \\ &= 50 \cdot 8^{\frac{4}{3}} \\ &= 50 \cdot 2^4 \\ &= 800 \end{aligned}$$

Bonus: Differentiate the function, $f(x) = x^x$ (assume $x > 0$).

Solution: $f(x) = x^x = e^{x \ln x}$, so

$$\begin{aligned} f'(x) &= e^{x \ln x} \frac{d}{dx} (x \ln x) \\ &= e^{x \ln x} (1 + \ln x) \\ &= x^x (1 + \ln x) \end{aligned}$$