Math 165 (Chris Kurth)
Spring 2008
Quiz 6

Show all work. Answers without work will not receive credit.

1. (10 points) Find the critical points of $f(x) = \frac{1}{2}x + \cos x$ where $0 < x < 2\pi$. Determine which are local maxima or local minima using one of the tests.

   Solution: $f'(x) = \frac{1}{2} - \sin(x)$, so $0 = f'(x)$ when $\sin(x) = \frac{1}{2}$, i.e. when $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. Since $f''(x) = -\cos(x)$ we have $f''(\frac{\pi}{6}) < 0$ and $f(\frac{5\pi}{6}) > 0$, hence $\frac{\pi}{6}$ is a local maximum and $\frac{5\pi}{6}$ is a local minimum.

2. (10 points) Let $f(x) = \dfrac{1}{4+x^2}$ be defined on the interval $[0, \infty)$. Find the global maximum and minimum or tell me if either one does not exist.

   Solution: $f'(x) = \dfrac{-2x}{(4+x^2)^2}$ which is only 0 when $x = 0$ (i.e. this is the only critical point). Also, $f(x)$ is decreasing to the right of 0, so there must be a global maximum at $x = 0$. On the other hand, there can be no global minimum, because any global minimum would also be a critical point, and the only critical point was at 0.

3. (Bonus) Let $f(x) = \dfrac{1}{x} + x$. Then $f'(x) = \dfrac{-1}{x^2} + 1$, so $f'(x) = 0$ only when $x = 1$ or $x = -1$. And $f(1) = 2$ and $f(-1) = -2$. Can I conclude that the global maximum of $f(x)$ on the closed interval $[-1, 1]$ is at $x = 1$? Why or why not?

   Solution: No. $f(x)$ is not continuous at 0 so the Max-Min Existence Theorem does not apply. In fact, $f(1/2) = 5/2 > 2 = f(1)$.