

Math 165 (Chris Kurth)

Spring 2008

Quiz 4

Show all work. Answers without work will not receive credit.

1. (6 points) Find the second derivative of

$$f(x) = \sqrt[3]{x} + \sqrt{x} + x + 1$$

Solution: $f''(x) = -\frac{2}{9}x^{-5/3} - \frac{1}{4}x^{-3/2}$

2. (7 points) A snowball in the shape of a sphere is melting so that the radius is decreasing at a uniform rate of 1 centimeter per second. How fast is the volume decreasing when the radius equals 6 centimeters?

Solution: $V = \frac{4}{3}\pi r^3$, and $\frac{dr}{dt} = -1 \frac{\text{cm}}{\text{sec}}$. Implicitly differentiate on both sides to get $\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$. Then when $r = 6\text{cm}$, $\frac{dV}{dt} = 4\pi \cdot (6\text{cm})^2 (-1 \frac{\text{cm}}{\text{sec}}) = -144 \frac{\text{cm}^3}{\text{sec}}$

3. (7 points) Using implicit differentiation, find $D_x y$ (also called $\frac{dy}{dx}$), where:

$$x^2 y^2 + 4xy = 12y$$

Solution: Differentiate with respect to x on both sides:

$$\begin{aligned} \frac{d}{dx}(x^2 y^2 + 4xy) &= \frac{d}{dx}(12y) \\ 2xy^2 + 2x^2 y \frac{dy}{dx} + 4y + 4x \frac{dy}{dx} &= 12 \frac{dy}{dx} \\ (2x^2 y + 4x - 12) \frac{dy}{dx} &= -4y^2 - 2xy^2 \\ \frac{dy}{dx} &= \frac{-4y - 2xy^2}{2x^2 y + 4x - 12} \end{aligned}$$

4. (Bonus) Find the equation of the tangent line to the curve from part 3 at $(2, 1)$.

Solution: Plug $x = 2$ and $y = 1$ into the result of part 3 to get $\frac{dy}{dx} = -2$ at $(2, 1)$. Thus the tangent line is:

$$y - 1 = -2(x - 2)$$