Math 165 (Chris Kurth)
Spring 2008
Quiz 4

**Show all work. Answers without work will not receive credit.**

1. (5 points) Evaluate:

$$\frac{d}{dx}\left(\frac{1}{(x^2-x+1)^7}\right)$$

Solution: $\frac{1}{(x^2-x+1)^7} = f(g(x))$ where $g(x) = x^2-x+1$ and $f(u) = u^{-7}$. Then $g'(x) = 2x-1$ and $f'(u) = -7u^{-8}$. So:

$$\begin{aligned} \frac{d}{dx}\left(\frac{1}{(x^2-x+1)^7}\right) &= \frac{d}{dx}\left(f(g(x))\right) \\ &= -7(x^2-x+1)^{-8}(2x-1) \end{aligned}$$

2. (5 points) Let $f(x) = \cos^6(\sin(x))$. Find $f'(x)$.

Solution: $f(x) = g(h(x))$ where $g(u) = \cos^6(u)$ and $u = h(x) = \sin(x)$. We have $g'(u) = 6\cos^5(u)\cdot(-\sin(u))$ and $h'(x) = \cos(x)$. So

$$f'(x) = g'(h(x))h'(x) = -6\cos^5(\sin(x))\sin(\sin(x))\cos(x)$$

3. (5 points) Express $D_x(F(x^2+1))$ in terms of $F(x)$.

Solution: Let $g(x) = x^2+1$. Then

$$\begin{aligned} D_x(F(x^2+1)) &= D_x(F(g(x))) \\ &= F'(g(x))g'(x) \\ &= F'(x^2+1)\cdot 2x \end{aligned}$$

4. (5 points) Find the equation of the tangent line to $y = \cot x$ at $(\frac{\pi}{4}, 1)$.

Solution: The tangent line to $y = f(x)$ at $(x_0, y_0)$ is

$$y - y_0 = m(x - x_0)$$

where $m = f'(x_0)$. We have $f(x) = \cot(x)$, so $f'(x) = -\csc^2 = \frac{-1}{\sin^2(x)}$, so $m = \frac{-1}{\sin^2(\pi/4)} = \frac{-1}{1/2} = -2$. And the tangent line at $(\frac{\pi}{4}, 1)$ is

$$y - 1 = -2(x - \pi/4)$$