Math 165 (Chris Kurth)
Spring 2008
Quiz 3

Show all work. Answers without work will not receive credit.

1. (6 points) $f(x) = \dfrac{x^2 - 5x - 6}{x - 6}$ is not defined at a certain point. How should it be redefined to make it continuous there?

   Solution: $f(x) = \dfrac{x^2 - 5x - 6}{x - 6} = \dfrac{(x-6)(x+1)}{x-6} = x + 1$ except at $x = 6$. So redefining

$$f(6) = \lim_{x \to 6} f(x) = 7$$

   gives a function continous everywhere.

2. (6 points) For what values of u is the following function continuous?

$$f(u) = \frac{2u + 7}{\sqrt{u + 5}}$$

   Solution: For the square root to be defined we must have $u + 5 \geq 0$, i.e. $u \geq -5$. For the denominator to be nonzero we must have $\sqrt{u+5} \neq 0$, i.e. $u \neq -5$. Hence $f(u)$ is continuous for $u > -5$.

3. (8 points) Using the limit definition, find the slope of the tangent line to the curve $y = x^3 - 3x$ at the point where $x = -2$.

   Solution:

$$\begin{aligned}
\text{slope} &= \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h} \\
&= \lim_{h \to 0} \frac{((-2+h)^3 - 3(-2+h)) - ((-2)^3 - 3(-2))}{h} \\
&= \lim_{h \to 0} \frac{-8 + 12h + -6h^2 + h^3 + 6 - 3h + 8 - 6}{h} \\
&= \lim_{h \to 0} \frac{9h + -6h^2 + h^3}{h} \\
&= \lim_{h \to 0} (9 + -6h + h^2) \\
&= 9
\end{aligned}$$