Math 165 (Chris Kurth)
Spring 2008
Quiz 4

Show all work. Answers without work will not receive credit.

1. Calculate the following limits:

   (a) $\lim_{x \to -1} \dfrac{x^2 - 2x - 3}{x + 1}$

   Solution:
   $$\lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1} = \lim_{x \to -1} (x - 3) = -4$$

   (b) $\lim_{x \to \infty} \dfrac{3x^7 + 4x^6 - 2x^3 + 4}{2x^7 - 3x^5 + x + 11}$

   Solution:
   $$\begin{aligned} \lim_{x \to \infty} \frac{3x^7 + 4x^6 - 2x^3 + 4}{2x^7 - 3x^5 + x + 11} &= \lim_{x \to \infty} \frac{3 + \frac{4}{x} - \frac{2}{x^4} + \frac{4}{x^7}}{2 - \frac{3}{x^2} + \frac{1}{x^6} + \frac{11}{x^7}} \\ &= = \frac{3 + 0 - 0 + 0}{2 - 0 + 0 + 0} \\ &= = \frac{3}{2} \end{aligned}$$

   (c) $\lim_{x \to 0} \dfrac{\sin 2x}{\cos x}$

   Solution: $\lim_{x \to 0} \dfrac{\sin 2x}{\cos x} = \dfrac{\sin 0}{\cos 0} = \dfrac{0}{1} = 0$

2. A ball is shot in the air and its height at time t is given by the function $h(t) = 96t - 16t^2$ where height is measured in feet. What is its velocity 2 seconds after launch?

   Solution: $h'(t) = 96 - 32t$, so $h'(2) = 32 \frac{\text{ft}}{\text{sec}}$

3. Define a function $f(x)$ by:
   $$f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$$

   As specifically as possible, evaluate:

   (a) $f(1) =$

   Solution: $f(1) = 2$

   (b) $\lim_{x \to 1^+} f(x) =$

   Solution: $\lim_{x \to 1^+} f(x) = 2$

(c) $\lim_{x\to 1^-} f(x) =$

Solution: $\lim_{x\to 1^-} f(x) = 1$

(d) $\lim_{x\to 1} f(x) =$

Solution: $\lim_{x\to 1} f(x)$ is not defined.

(e) $\lim_{x\to 0^+} f(x) =$

Solution: $\lim_{x\to 0^+} f(x) = \infty$

4. Find the derivative of each function:

(a) $f(x) = 3x^5 + 7x^2 + 32 + x^{-1}$

Solution: $f'(x) = 15x^4 + 14x - x^{-2}$

(b) $f(x) = (x+3)(2x^2 - 5)$

Solution: $f'(x) = (x+3)\cdot(4x) + (2x^2-5)\cdot 1 = 6x^2 + 12x - 5$

(c) $f(x) = \dfrac{x}{x^2+x+1}$.

Solution: $f'(x) = \dfrac{(x^2+x+1)\cdot 1 - x(2x+1)}{(x^2+x+1)^2} = \dfrac{-x^2+1}{(x^2+x+1)^2}$

5. **Use the definition of the derivative** to calculate $f'(x)$ where

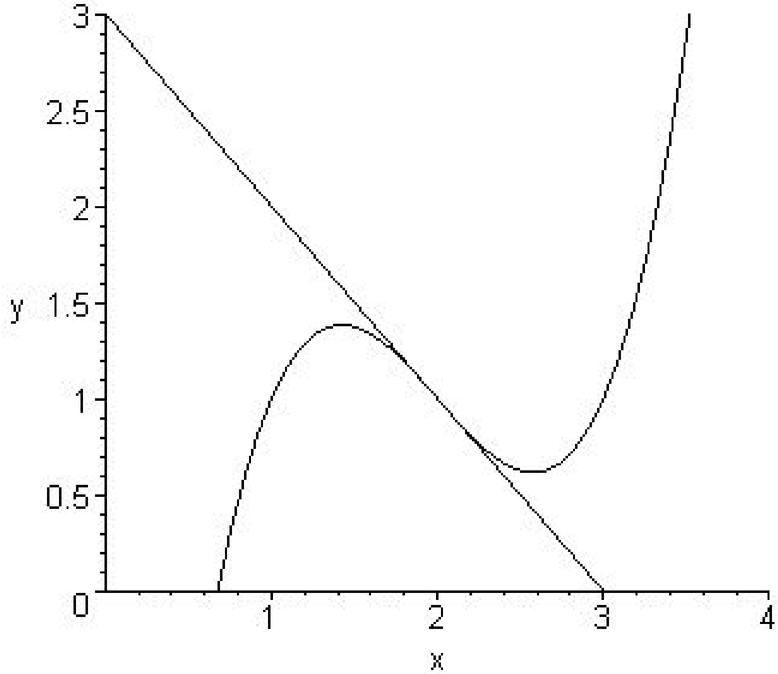
$$f(x) = \frac{3}{x}$$

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} \\
&= \lim_{h\to 0} \frac{\frac{3x-3(x+h)}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{-3h}{hx(x+h)} \\
&= \lim_{h\to 0} \frac{-3}{x(x+h)} \\
&= \frac{-3}{x^2}
\end{aligned}$$

6. Sketch the tangent line to the graph of $f(x)$ at $x = 2$. Is the derivative positive, negative or zero there?

Solution: The diagram looked like:



The derivative is negative at $x = 2$.