Problem 1.

(A) From the SAS MIXED procedure: $\hat{\sigma}_\tau^2 = 92.8730, \hat{\sigma}_e^2 = 22.0791$.

(B) H_0:there is no drug and disease interaction.
By the output from PROC MIXED, F=3.93 with (6,41) df. p-value=0.0034, so we reject the null hypothesis and conclude the drug and disease interaction is significant.

(C) In this case, PROC GLM produces almost the same F-value as PROC GLM (F=3.91 with (6,41) df and p-value=0.0036). Both F-values lead us to reject the null hypothesis and conclude the drug and disease interaction is significant. The test provided by PROC MIXED is more appropriate to use. In this problem, PROC GLM use least squares estimation to estimate the fixed effects parameters in the model. The least squares estimates are linear unbiased estimators, but they are not best linear unbiased estimators because observations on the same dog are correlated and the number of observations is not the same for all combinations of factors. PROC MIXED uses the REML estimates of the variance components to compute approximate generalized least squares estimators of the fixed effects parameters which have slightly smaller variances than the least squares estimators produced by PROC MIXED. Consequently, the F-test provided by PROC MIXED has slightly greater power than the F-test provided by PROC GLM and this is evidenced by the slightly larger F-value produced by PROC MIXED.

(D) For each disease we have an F-test of the null hypothesis that the mean results are the same for all four drugs. The results for the F-test are as follows:

Disease	Numerator df	Denominator df	F-value	p-value
1	3	41	10.19	<.0001
2	3	41	27.71	<.0001
3	3	41	6.83	.0008

These F-test indicate that there are differences in mean increase in blood pressure for at least tow of the drugs for each disease. To determine which drugs provide different mean increases in blood pressure for a particular disease you must further investigate estimates of mean results for the four drugs within each disease. You can use the LSMEANS option in PROC MIXED of the code for comparing means included in the S-PLUS code posted on the course web page.

For disease 1, drugs 1 and 2 provide higher mean increases in systolic blood pressure than drugs 3 and 4, and there is no significant difference between drug 1 and 2 or between drug 3 and 4 in pairwise comparisons. For disease 2, drug 2 provides the highest mean increase in blood pressure and drug 3 provides the lowest increase in blood pressure. For disease 3, there is no significant difference between any 2 drugs in pairwise comparisons. Overall speaking, drug 3 provides the lowest mean increase in blood pressure for each disease.

(E)

clinic	1	2	3	4	5	6
effect	10.59	6.20	5.26	-15.69	-5.15	-1.21

You can plot these values on a normal probability plot, but it is difficult to assess the normality assumption because we have only have information on 6 clinic. There is no indication that the clinic effects are not normally distributed.

(F) From the normal probability plot, the residuals appear to be a random sample from a normal distribution.

Problem 2.

(A) ANOVA table:

source	DF	SS	MS	F	p-value
method	2	1.9489	0.9744	20.01	<0.0001
plants	3	343.31	114.43	4.89	<0.05
leaves(plants)	8	186.90	23.36	479.84	<0.0001
error	22	1.0711	0.04869		

(B) Formulas for expectations of mean squares:

source	expected mean squares
plants	$\sigma_e^2 + 3\sigma_\gamma^2 + 9\sigma_\beta^2$
leaves(within plants)	$\sigma_e^2 + 3\sigma_\gamma^2$
method	σ_e^2 + (quadratic form involving method effects)
error(within leaves)	σ_e^2

(C) REML estimates for the variance components:
$\hat{\sigma}_\beta^2 = 10.1193,\ \hat{\sigma}_\gamma^2 = 7.7711,\ \hat{\sigma}_e^2 = 0.04869.$
The largest source of random variation is from plant variation.

(D) $\bar{Y}_{3..} = 14.5917$
$E(\bar{Y}_{3..}) = \mu + \alpha_3$
$var(\bar{Y}_{3..}) = 1/12(\sigma_e^2 + \sigma_\gamma^2 + 3\sigma_\beta^2)$
s.d of $\bar{Y}_{3..} = \sqrt{1/12(\hat{\sigma}_e^2 + \hat{\sigma}_\gamma^2 + 3\hat{\sigma}_\beta^2)} = 1.7837$
A 95% CI for $\mu + \alpha_3$ is: $14.5917 \pm t_{3.01,.975} 1.7837 = [8.9152, 20.2682]$
where 3.01 is the Cochran-Satterhwaite degrees of freedom.

(E) $E(\bar{Y}_{3..} - \bar{Y}_{1..}) = \alpha_3 - \alpha_1$
$\bar{Y}_{3..} - \bar{Y}_{1..} = 14.5917 - 14.0750 = 0.5167.$
$var(\bar{Y}_{3..} - \bar{Y}_{1..}) = \frac{1}{6}\sigma_e^2$
s.d. of $\bar{Y}_{3..} - \bar{Y}_{1..} = \sqrt{\frac{1}{6}\hat{\sigma}_e^2} = 0.0901$

A 95% CI for $\alpha_3 - \alpha_1$ is: $0.5167 \pm t_{22,0.975} * 0.0901 = [0.3299, 0.7035]$

(F) The mean acid concentration measurement provided by method C is significantly higher than mean acid concentrations measurement provided by methods A and B(adjusted Tukey p-values are less than 0.0001 in both comparisons), but there is no significant difference between mean acid concentrations provided by methods A and B (p-value=0.845).

(G) $corr(Y_{ijk}, Y_{sjm}) = \frac{cov((Y_{ijk}, Y_{sjm})}{\sqrt{var(Y_{ijk} var Y_{sjm}}} = \frac{\sigma_\beta^2}{\sigma_e^2 + \sigma_\beta^2 + \sigma_\gamma^2}$ is estimated as $\hat{\rho} = 10.1193/17.939 = 0.564$

(H) $corr(Y_{ijk}, Y_{sjk}) = \frac{cov(Y_{ijk}, Y_{sjk})}{\sqrt{var(Y_{ijk} var(Y_{sjk})}} = \frac{\sigma_\beta^2 + \sigma_\gamma^2}{\sigma_e^2 + \sigma_b^2 eta + \sigma_\gamma^2}$ is estimated as $\hat{\rho} = 17.890/17.939 = 0.997$

(I) The new model is: $Y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$, where Y_{ij} is the observed acid concentration made by the i-th method on a leaf from the j-th plant, $\beta_j \sim NID(0, \sigma_\beta^2)$, $\epsilon_{ij} \sim NID(0, \sigma_\epsilon^2)$, and $\sigma_\epsilon^2 = \sigma_\gamma^2 + \sigma_e^2$. The new experiment would result in less precise comparisons of the three methods, because different methods are applied to different leaves instead of being applied to the same leaf. Variability in the difference between estimated means for any pair of methods now involves variation among leaves in addition to random measurement error. In estimating the difference between the mean acid concentrations determined by two methods, we now have

$$var(\bar{Y}_{i..}) - var(\bar{Y}_{k..}) = \frac{1}{6}\sigma_\epsilon^2 = \frac{1}{6}(\sigma_\gamma^2 + \sigma_e^2)$$

For the original experiment (see part E) we have

$$var(\bar{Y}_{i..}) - var(\bar{Y}_{k..}) = \frac{1}{6}\sigma_e^2$$

The ratio of these two variances is the relative efficiency of the two experiments with respect to the estimation of differences in mean acid concentration determined by two different methods, i.e.

$$\text{efficiency of the new experiment} = \frac{\sigma_\gamma^2 + \sigma_e^2}{\sigma_e^2}$$

Substituting the estimates of the variance components obtained in part C, the estimated efficiency of the new experiment relative to the original experiment is $.04869/(7.7711 + .04869) = .0062265$. Consequently, the number of observations made on each method in the new experiment would have to be about $1/(.0062265)$=160 times greater than the number of observations made in the original experiment to estimate the difference in mean acid concentration by provide by two different methods with about the same accuracy.

Problem 3.

(A) The experimental units are the pigs. the random blocking factor is litter, the levels of the fixed treatment factor are the three methods for controlling worms.

(B) $Y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$ where Y_{ij} is the amount of weight gained made by the pig from the j-th litter assigned to the j-th method for controlling worms, $\beta_j \sim NID(0, \sigma_\beta^2)$, $\epsilon_{ij} \sim NID(0, \sigma_\epsilon^2)$ and any β_j is independent of any $\epsilon_(ij)$.

(C) ANOVA table:

source	df	expectation of mean squares
litters	9	$\sigma_e^2 + 3\sigma_\beta^2$
treatments2	$\sigma_e^2 + 5\sum_{i=1}^{3}\alpha_i^2$	
error(pigs within litters)	18	σ_e^2

(D) $\bar{Y}_{1.} = \frac{1}{10}\sum_{j=1}^{10} Y_{1j}$ and $var(\bar{Y}_{1.}) = \frac{1}{10}(\sigma_\beta^2 + \sigma_e^2)$
Hence, an estimate of $var(\bar{Y}_{1.})$ is $S^2_{\bar{Y}_{1.}} = \frac{1}{10}(MSE + \frac{MS_{litter} - MSE}{3})\frac{1}{10}(\frac{2}{3}MSE + \frac{1}{3}MS_{litter})$
A 95% CI for the mean weight gain for treatment A is $\bar{Y}_{1.} \pm t_{\nu,.975}S_{\bar{Y}_{1.}}$
where $\nu = \frac{S^2_{\bar{Y}_{1.}}}{(\frac{1}{15}MSE)^2/18 + (\frac{1}{30}MS_{litter})^2/9}$

(E) $E(\bar{Y}_{1.} - \bar{Y}_{3.}) = \alpha_1 - \alpha_3$ and $var(\bar{Y}_{1.} - \bar{Y}_{3.}) = \frac{2}{10}\sigma_e^2$
Hence, the standard error of $\bar{Y}_{1.} - \bar{Y}_{3.}$ is $\sqrt{MSE/5}$ and a 95% CI for $\alpha_1 - \alpha_3$ is:

$$(\bar{Y}_{1.} - \bar{Y}_{3.}) \pm t_{18,.975}\sqrt{MSE/5}$$

Problem 4.

(A) Fixed blocking factors: none
Random blocking factors: autos and drivers
Fixed treatment factors: gas additives
Random treatment factors: none

(B) $Y_{ijk} = \mu + \alpha_i + \beta_j + \tau_k + \epsilon_{ijk}$
where Y_{ijk} is the oxide level produced by additive i when the k-th car is driven by the j-th driver, and $\beta_j \sim NID(0, \sigma_\beta^2)$, $\tau_k \sim NID(0, \sigma_\tau^2), \epsilon_{ijk} \sim NID(0, \sigma_e^2)$, and any β_j, τ_k, ϵ_{ijk} are mutually independent.

(C) ANOVA table:

source	df	SS	MS	F-value	p-value	E(MS)
additive	3	40	13.33	5	0.0452	var(error)+Q(additive)
auto	3	24	8	3	0.1170	var(error)+4var(auto)
driver	3	216	72	27	0.0007	var(error)+4var(driver)
error	6	16	2.67			var(error)

(D) $\hat{\sigma}^2_\beta = 17.3333, \hat{\sigma}^2_\tau = 1.3333, \hat{\sigma}^2_e = 2.6667$
Since it is a balanced design, the REML estimates are equal to method of moments estimators (check it).

(E) $\bar{Y}_{1..} = \frac{1}{4}\sum_{j=1}^{4}\sum_{k=1}^{4} Y_{1jk} = 18$ and $var(\bar{Y}_{1..}) = \frac{1}{4}(\sigma^2_\beta + \sigma^2_\tau + \sigma^2_e)$
The stadard error for $\bar{Y}_{1..}$ is: $S_{\bar{Y}_{1..}} = \sqrt{(\hat{\sigma}^2_\beta + \hat{\sigma}^2_\tau + \hat{\sigma}^2_e)/4} = \sqrt{\frac{2}{4}MSE + \frac{1}{4}MS_{auto} + \frac{1}{4}MS_{driver}} = 2.3094$

A 95% CI for the mean oxide reduction by method A is:
$18 \pm t_{\nu,.975} * 2.3094 = [11.6792, 24.3208]$
where ν=4.15 by Cochran-Satterthwaite approximation.

(F) $\bar{Y}_{1..} - \bar{Y}_{2..} = 18 - 22 = -4$ and $var(\bar{Y}_{1..} - \bar{Y}_{2..}) = \frac{2}{4}\sigma^2_e$
The standard error for$\bar{Y}_{1..} - \bar{Y}_{2..}$ is: $\sqrt{MSE/2} = 1.1547$
A 95% CI for the difference in the mean oxide reductions by methods A and B is:
$-4 \pm t_{6,.975} * 1.1547 = [-6.8255, -1.1745]$

(G) HSD shows that method B provides significantly greater oxide reduction than either method A or D. There is no significant difference in mean oxide reduction between methods B and C. Methods A and D can be eliminated from future consideration. Any additional runs should examine the difference between methods B and C.

Problem 5.

(A) primary (or whole plot) experimental units: trays
sub-plot units: pots
treatment factors: levels of fertilizers and moisture
blocking factors: none

(B) ANOVA table:

source	df	SS	MS	F	p-value
moisture	3	269.19	89.73	26.34	0.0002
trays(moisture)	8	27.25	3.406	4.53	0.0019
fertilizer	3	297.05	99.018	131.65	<0.0001
moist*fert	9	38.06	4.228	5.62	0.0003
error	24	18.05	0.752		

	expected mean squares
moisture	var(error)+4var(tray)+Q(moist,moist*fert)
tray	var(error)+4var(tray)
fertilizer	var(error)+Q(ferti,moist*ferti)
moist*fert	var(error)+Q(moist*ferti)
error	var(error)

(C) Method of moments estimates of variance components are: $\hat{\sigma}^2_e = 0.7521, \hat{\sigma}^2_\gamma = 0.6635$

(D) $\mu + \tau_1$ is not estimable.
$\mu + \alpha_1 + \tau_1 + \delta_{11}$ is estimable since
$E(\bar{Y}_{1.1}) = \mu + \alpha_1 + \tau_1 + \delta_{11}$
$\tau_1 - \tau_2$ is not estimable.
$\alpha_1 + \delta_{11} - \alpha_2 - \delta_{21}$ is estimable since
$E(\bar{Y}_{1.1} - \bar{Y}_{2.1}) = \alpha_1 + \delta_{11} - \alpha_2 - \delta_{21}$
$\delta_{11} - \delta_{13} - \delta_{21} + \delta 23$ is estimable since
$E(\bar{Y}_{1.1} - \bar{Y}_{1.3} - \bar{Y}_{2.1} + \bar{Y}_{2.3} = \delta_{11} - \delta_{13} - \delta_{21} + \delta 23$
$(\alpha_1 + \frac{1}{4}\sum_{k=1}^{4} \delta_{1k}) - (\alpha_2 + \frac{1}{4}\sum_{k=1}^{4} \delta_{2k})$ is estimable since
$E(\bar{Y}_{1..} - \bar{Y}_{2..}) = (\alpha_1 + \frac{1}{4}\sum_{k=1}^{4} \delta_{1k}) - (\alpha_2 + \frac{1}{4}\sum_{k=1}^{4} \delta_{2k})$
By the output from PROC MIXED, A 95% CI for $\mu + \alpha_1 + \tau_1 + \delta_{11}$ is :
$3.122 \pm t_{19.3,.975} * 0.687 = [1.6856, 4.5584]$
A 95% CI for $\alpha_1 + \delta_{11} - \alpha_2 - \delta_{21}$ is:
$-2.8491 \pm t_{19.3,.975} * 0.9715 = [-4.8804, -0.8177]$
A 95% CI for $\delta_{11} - \delta_{13} - \delta_{21} + \delta 23$ is:
$2.6673 \pm t_{24,.975} * 1.0014 = [0.6004, 4.7341]$
A 95% CI for $(\alpha_1 + \frac{1}{4}\sum_{k=1}^{4} \delta_{1k}) - (\alpha_2 + \frac{1}{4}\sum_{k=1}^{4} \delta_{2k})$ is:
$-5.0510 \pm t_{8,.975} * 0.7535 = [-6.7885, -3.3135]$

(E) The profile plot with moisture level on the horizontal axis shows a quadratic trend for each fertilizer level. There is no indication of existence of interaction.
The profile plot with fertilizer level on the horizontal axis shows a linear trend for each fertilizer level. There is no strong evidence of existence of interaction.
REML estimates for σ_e^2 and σ_γ^2:
$\hat{\sigma}_e^2 = 1.4407, \hat{\sigma}_\gamma^2 = 0.3968$

effect	estimate	s.d.	df	t	p-value
β_0	10.5489	0.4567	14.6	23.10	<0.0001
β_1	0.1294	0.02246	9	5.76	0.0003
β_2	1.1066	0.07748	33	14.28	<0.0001
β_3	0.01818	0.00693	33	2.62	0.0131
β_4	-0.01875	0.002512	9	-7.46	<0.0001
β_5	0.04888	0.04331	33	1.13	0.2672

F From the output,the estimate of mean weight is 7.38 and A 95% CI is $[6.46, 8.30]$

(G) Because time trends and difference in time trends are within tray contrasts and comparisons, we can obtain an approximate F-test of the null hypothesis:

H_0: the reduced model is appropriate

by comparing residual sums of squares. The ANOVA table for the model in part E:

source	df	SS
model	14	602.06
error	33	47.54

We can perform the lack of fit test:
$F = \frac{(SSE(reduced) - SSE(full))/(33-24)}{SSE(full)/24} = \frac{(47.54-18.05))/(33-24)}{18.05/24} = 4.36$ on (9,24) df with p-value=0.002
So we reject the null hypothesis and conclude the reduced model is not appropriate.

This F-test is not entirely appropriate because the estimates of the variance components change for the two models (this invovles the variation among trays in addition to the within tray random variation). It would be better to perform a likelihood ratio test. This can be done with PROC MIXED in SAS bat adding the method=ml option to the PROC MIXED statement. It can be done in S-PLUS by adding the argument method="ML" to the lme( ) function. SAS prodcues the value of -2(log-likelihood) and S-PLUS produces the value of the log-likelihood simultaneously fitting both the fixed effects and the variance components. The results are:

model	log-likelihood	-2(log-likelihood)
general effects	-60.60	121.2
quadratice surface	-78.04	156.1

The value of the chi-square test of the null hypothesis that the quadratic surface fits the data as well as the general effects model is

$$\chi^2 = 156.1 - 121.2 = 34.9 \text{ with 10 df and p-value¡.0001}$$

The proposed quadratic surface is not adequate, look for a better model.