

Practice Problems; Review HW6-8 and Quiz6-8 also.

- (1)  $\lim_{x \rightarrow 0} \frac{\arctan(x) - x}{x^3}$
- (2)  $\lim_{x \rightarrow 0^+} x \ln(x)$
- (3)  $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{3x}$
- (4)  $\lim_{x \rightarrow 0^+} \frac{\int_0^x \sqrt{t} \tan(\sqrt{t}) dt}{x^2}$
- (5) Find the area of the region bounded above by  $f(x) = \frac{1}{(x+2)^2}$ , below by the  $x$ -axis, and on the left by  $x = -1$ . The region is unbounded on the right.
- (6) Evaluate the following improper integrals or show that they diverge.
  - (a)  $\int_1^{\infty} \frac{x}{(1+x^2)^2} dx$
  - (b)  $\int_0^2 \frac{1}{(x-1)^2} dx$
- (7) Express the repeating decimal  $0.325325 \dots$  as a geometric series, and find its value as rational number.
- (8) A ball is dropped from a height of 100 feet. Each time it hits the floor, it rebounds to  $2/3$  its previous height. Find the total distance it travels before coming to rest.
- (9) Show that  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$
- (10) Show that  $S = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}}$  converges and find a bound for  $|S - S_{99}|$
- (11) Determine whether the following series converge or diverge.
  - (a)  $\sum_{n=2}^{\infty} \frac{1}{n[\ln n]^2}$
  - (b)  $\sum_{n=1}^{\infty} \left(-\frac{1}{5}\right)^{-n}$
  - (c)  $\sum_{n=1}^{\infty} \left(\frac{1}{7^n} + \frac{1}{n^7}\right)$
  - (d)  $\sum_{n=1}^{\infty} \frac{1}{1+3^{-n}}$
  - (e)  $\sum_{n=1}^{\infty} \frac{n^{3/2}}{n^{5/2}+1}$  (Use the integral test)
  - (f)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{\sqrt{n}}$
  - (g)  $\sum_{n=1}^{\infty} \frac{2^n}{(n+1)!}$
  - (h)  $\sum_{n=1}^{\infty} \{\arctan(n+1) - \arctan(n)\}$
  - (i)  $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$
  - (j)  $\sum_{n=1}^{\infty} \frac{6n}{3n^3+1}$
  - (k)  $\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$

$$(l) \sum_{n=1}^{\infty} \frac{n^n}{(n+1)!}$$

$$(m) \sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$$

(12) Classify the following series as divergent, conditionally convergent, or absolutely convergent series.

$$(a) \sum_{n=1}^{\infty} \frac{(-5)^n}{6^n}$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$$

$$(c) \sum_{n=1}^{\infty} \sin \frac{\pi}{2} n$$

$$(d) \sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n\sqrt{n}}$$

$$(e) \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$$