Math 265, Section E2 - Test 2

10 March 2006 Name: ____________________________

DIRECTIONS: Answer the following questions or execute the following commands below. You may NOT use a calculator. Remember, you are an attorney and I am a jury of 12 people. You must convince me beyond a reasonable doubt that your answers are correct by showing work and *writing neatly*. If something is not clear, notify me immediately and I shall elucidate. Should you have any other questions, do not hesitate to ask them.

(15 points) 1. Let $f(x, y, z) = x^2 + 2y^2 + 3z^2$.

(a) Find the equation for the level surface of f at the point $(1, 2, 3)$.

(b) What is the name for the quadric surface you just found?

(c) Compute the equation for the plane tangent to the surface found in (a) at the point $(1, 2, 3)$.

(10 points) 2. Find the directional derivative of $f(x, y) = e^{-xy}$ at the point $(1, -1)$ in the direction of the vector $-\mathbf{i} + \sqrt{3}\mathbf{j}$.

(10 points) 3. Decide whether each of the following statements is true or false (you don't have to justify your choice here).

(a) If a function of 3 variables is differentiable at a point $\mathbf{p}$, then it is also continuous at the point $\mathbf{p}$.

(b) The gradient of f at a point P is tangent to the level curve of f that passes through the point P.

(c) In the Second Partials Test for a function f of two variables, the number D (as stated in the theorem) plays a crucial role.

(d) For all differentiable functions f and g, $\nabla [f(\mathbf{p})g(\mathbf{p})] = f(\mathbf{p})\nabla g(\mathbf{p}) + g(\mathbf{p})\nabla f(\mathbf{p})$.

(e) A function increases most rapidly at $\mathbf{p}$ in the direction of $|\nabla f(\mathbf{p})|$.

(15 points) 4. Find the extrema, if any, of the function F defined by $F(x, y) = 3x^3 + y^2 - 9x + 4y$.

(20 points) 5. Find the gradient for each of the following functions below:

(a) $f(x, y) = xe^{xy}$

(b) $f(x, y) = y^2 + \ln(yx)$

(c) $f(x, y, z) = x^2ye^{x-z}$

(d) $f(x, y, z) = x^2y + y^2z + z^2x$

(15 points) 6. Show that if

$$w = f(r - s, s - t, t - r)$$

then

$$\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = 0.$$

(15 points) 7. A function $f(x, y)$ is called *homogeneous of degree 1* if $f(tx, ty) = tf(x, y)$ for all $t > 0$.

(a) Are constant functions homogeneous? Give reasons for your answer.

(b) Prove **Euler's Theorem**; that such a function satisfies

$$f(x, y) = x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$$

[You may *not* use the fact that Euler's Theorem is true in part (a).]