Math 265, Section E2 - Test 1

3 February 2006 Name: _____

DIRECTIONS: Answer the following questions or execute the following commands below. You may NOT use a calculator. Remember, you are an attorney and I am a jury of 12 people. You must convince me beyond a reasonable doubt that your answers are correct by showing work and *writing neatly*. Should you have any questions, do not hesitate to ask them. There are questions on the back!

(20 points) 1. Let $\mathbf{u} = \langle 3, 2, 1 \rangle$, $\mathbf{v} = \langle 2, 1, 3 \rangle$, and $\mathbf{w} = \langle 1, 2, 3 \rangle$. Compute each of the following:

(a) $\mathbf{u} \cdot \mathbf{w}$

(b) $\mathbf{u} \cdot \mathbf{v}$

(c) $(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

(d) $(\mathbf{u} \cdot \mathbf{w})\mathbf{v}$

(e) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

(10 points) 2. Let a curve y be given by $y = 2x^2$. Find the curvature κ of y at the point $(1, 2)$.

(10 points) 3. If $\mathbf{F}(t) = (t^6)\,\mathbf{i} - (7e^{\cos t})\,\mathbf{j} + [\ln(\ln t)]\,\mathbf{k}$, find $\mathbf{F}'(t)$.

(10 points) 4. Determine whether each statement is true or false:

(a) For any vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$.

(b) For any vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

(c) For any vectors $\mathbf{u}$ and $\mathbf{v}$, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{v} \times \mathbf{u}|$.

(d) For any vectors $\mathbf{u}$ and $\mathbf{v}$, $(\mathbf{u} + \mathbf{v}) \times \mathbf{v} = \mathbf{u} \times \mathbf{v}$.

(e) For any vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$, $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$.

(10 points) 5. (a) State the perpendicularity criterion.

(b) State the parallel criterion.

(15 points) 6. Suppose a continuous function f has the property that

$$\int_0^x f(t)\ dt = xe^{2x} + \int_0^x e^{-t} f(t)\ dt$$

Find an explicit formula for $f(x)$.

(10 points) 7. Suppose four vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, and $\mathbf{d}$ all lie in the same plane. Prove that

$$(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = \mathbf{0}$$

Be as clear and concise as possible.

(15 points) 8. *Without using the cross product*, find two vectors that are perpendicular to both $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $-\mathbf{i}+2\mathbf{j}$ and have magnitude 5.