Math 265, Section E2 - Critical Thinking

March 3, 2006 Name: _____

DIRECTIONS: Here are some critical thinking questions. I will *never* show you how to do them, but I can give you advice on where to start. You are responsible for coming up with well-written solutions to these problems if you want to succeed in my class. I will likely add to this page whenever I feel the need, so try to stay up to date on this list of questions; i.e., please look at them before the next exam.

1. If $z = g(x)h(y)$, show that
$$\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = z\frac{\partial^2 z}{\partial x \partial y}$$

2. If $z = xe^{y/x}$, show that
$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

3. If $z = \ln\left(\sqrt{x^2+y^2}\right)$, show that
$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 1$$

4. If $w = f(x^2 - y^2, y^2 - x^2)$, show that
$$y\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial y} = 0$$

5. Show that
$$\nabla(f^r) = rf^{r-1}\nabla f$$
without using the chain rule.

6. Call a function $f(x, y)$ *homogeneous of degree* 1 if $f(tx, ty) = tf(x, y)$ for all $t > 0$. Prove **Euler's Theorem** that such a function satisfies
$$f(x, y) = x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$$