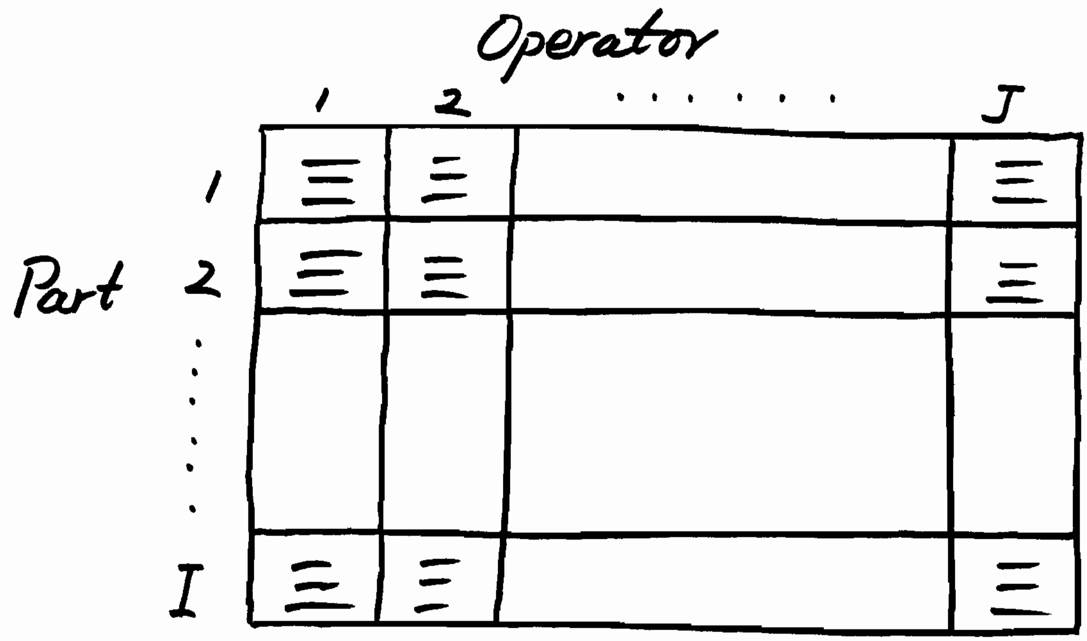
Balanced 2-way Factorial Data are typically used in Gage R&R contexts



Section 2.2.2 of V&J does range-based estimation of

$$\sigma$$

$$\sigma_{\text{reproducibility}} = \sqrt{\sigma_\beta^2 + \sigma_{\alpha\beta}^2}$$

$$\sigma_{\text{overall}} = \sigma_{R\&R} = \sqrt{\sigma^2 + \sigma_{\text{reproducibility}}^2}$$

A better method is to use 2-way ANOVA and the associated mean squares.

— Necessary facts are summarized in Section 1.4 of the notes.

Under Model (2.4) of V & J

$$MSA,\ MSB,\ MSAB,\ MSE$$

are independent sample-variance like objects and quantities of interest are again of form

$$\theta = g\left(E(MSA), E(MSB), E(MSAB), E(MSE)\right)$$

and natural point estimates are

$$U = g(MSA, MSB, MSAB, MSE).$$

Standard errors for such estimates can be made as before (using (1.3) of the notes) and Burdick & Graybill material can be used to make CI's.

— Andy Chiang has implemented many useful parts of this (for Gage R&R studies)

One important point about this material is that you will quickly learn that standard corporate forms for Gage R&R studies prescribe very bad experimental designs

— e.g., $I=10$, $J=2$ or 3 and $m=3$

Read the balance of Ch. 2 of V&J.
In particular, have a look at §2.2.3 and the impact of $\sigma_{\text{measurement}}$ on the ability to detect change or difference.

Leave the issue of assessing and quantifying measurement precision and talk a bit about statistics and improving measurement accuracy — §1.6 of the notes.

Application of polynomial regression to calibration

Calibration Experiment: measure several items with both a "perfect" measurement system and my current/local system of interest — (This could be: get standard specimens from NIST and measure locally) -

x = truth ⟵ known/constant

y = local measurement ← random

Standard polynomial regression model

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_k x^k + \underset{N(0,\sigma^2)}{\varepsilon}$$

β's and σ^2: unknown parameters

$(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$

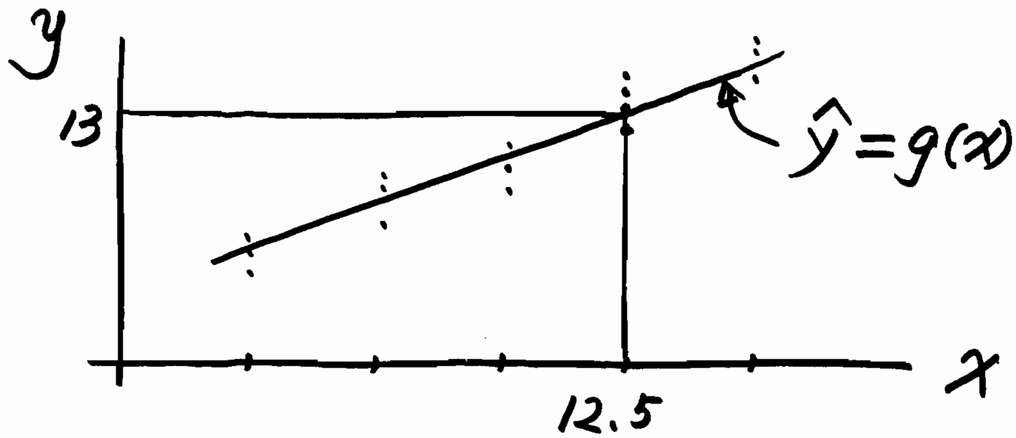
are typically used to do inference for unknown parameters.

Standard linear models theory tells us to do least squares, get fitted coefficients $b_0, b_1, \cdots, b_k$, and

$$\hat{y} = g(x) = b_0 + b_1 x + \cdots + b_k x^k.$$

In the present context this suggests that upon measuring y (locally) I should convert to x via

$$g^{-1}(y).$$



This is a "prescription" for point estimation of tomorrow's x from tomorrow's y — Confidence limits??

Recall that you are taught how to make prediction limits in standard methods courses —

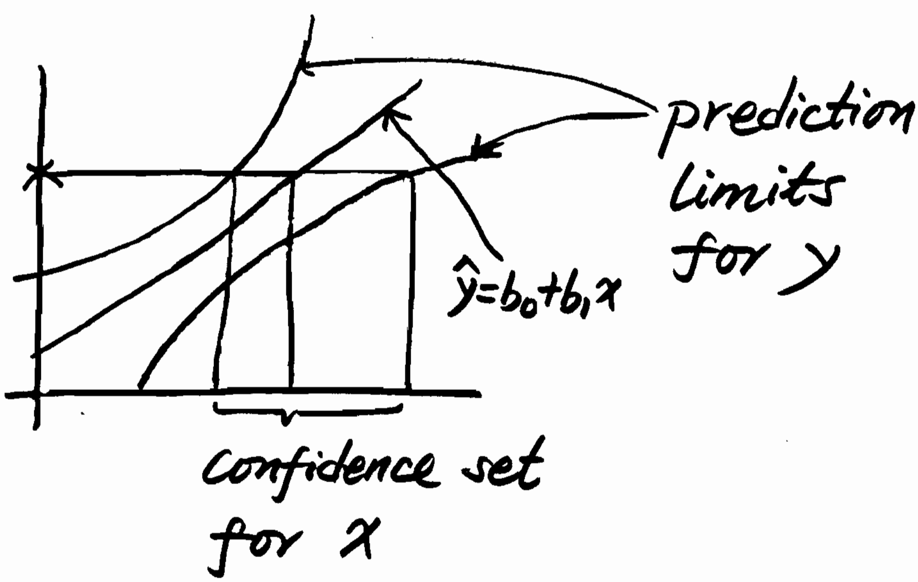
$$\hat{y} \pm t \cdot \sqrt{s^2 + (\text{standard error } \hat{y})^2}$$

— e.g., simple linear regression (SLR) version is

$$(b_0 + b_1 x) \pm t \cdot s \cdot \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$$

and I can "invert" prediction limits for y to get confidence sets for x

— In nice cases (SLR version)



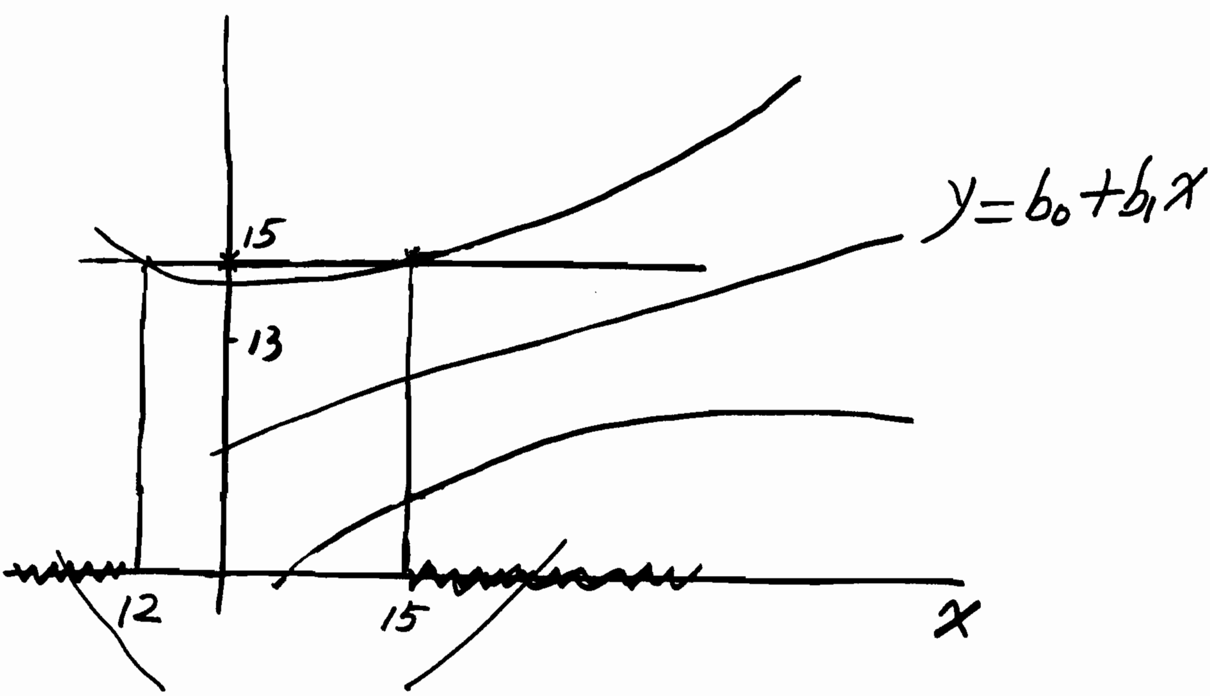
A confidence set for x_{n+1} based on the observed value y_{n+1} is

$$\{x \mid \text{the prediction interval for } y \text{ corresponding to } x \text{ includes } y_{n+1}\}$$

- This works pretty well in practice as long as n is large and σ is small.

- When σ is big and n is small, however, unpleasant things can happen.

  bad local measurements



Confidence set for x

Another set of issues regarding statistics and measurement is related to the question:

"What do I do about 'discreteness' in measurement brought on by relatively crude gaging?"

Fact: typical normal theory formulas implicitly suppose that numbers put into there are "exact"

"Exact": in the sense that when I write 1.213 inches I mean

1.21300000000 inches.

but in truth I typically mean

$$1.2125 < y < 1.2135$$

Q: Does this really matter?

A: It depends (primarily on how big σ is)

σ is big relative to the finest gradation in measurement $\longrightarrow$ no problem

On the other hand, if σ is small the standard confidence guarantees are void.

Vardeman's advice when gaging is crude relative to variation in quantity of interest:

— Don't pretend discrete things are cont$\underline{s}$ … treat them as discrete when

1) generating or thinking about appropriate distributions of statistics of interest

2) estimating parameters of underlying cont$\underline{s}$ dsns (seen only through crude gaging)

Discrete Observations and Distributions of Common Statistics

Suppose Y takes integer values, and let

$$f(y) = P[Y = y]$$

be the pmf for Y ← an individual measured value

(probability mass function)

Where is f from??

One possibility is sampling under a fixed set of conditions and using an empirical relative frequency dsn for f.

Suppose I have a sample of Y's (which I might model as iid from f)

What about dsns for common statistics of interest?

$\bar{Y}$, $\tilde{Y}$, s, R, etc. ($\tilde{Y}$: median)

One simple-minded way to approximate distributions of such statistics is through simulations –

y	$f(y)$
-1	.20
0	.55
1	.20
2	.05

$\overset{?}{\Longrightarrow}$ a probability dsn for $\bar{Y}$, $\tilde{Y}$, s, R based on $n=5$

For some standard statistics it is easy to see how to write algorithms to compute exact dsns

<u>For the sample mean, $\bar{Y}$:</u>

averages are essentially sums and dsns of sums come easily by adding on diagonals of joint probability tables

		$\nearrow -2$	$\nearrow -1$	$\nearrow 0$	
$y_1 \backslash y_2$	-1	0	1	2	
-1	.04	.11	.04	.01	.20
0	.11	.3025	.11	.0275	.55
1	.04	.11	.04	.01	.20
2	.01	.0275	.01	.0025	.05
	.20	.55	.20	.05	

So the pmf of $\bar{Y}$ $(n=2)$ is

$\bar{y}$	$P(\bar{Y}=\bar{y})$
-1	.04
$-.5$	.22
0	.3825
.5	.24
1	.095
1.5	.02
2	.0025

For $n=3$ think

$$Y_1+Y_2+Y_3=\underbrace{(Y_1+Y_2)}_{\text{dsn for this is already available}}+Y_3$$

For $n=4$:

$$Y_1+Y_2+Y_3+Y_4=(Y_1+Y_2)+(Y_3+Y_4)$$

Dsn of R (pages 12–13 of the Notes):

Without loss of generality (WOLOG).

suppose $f(y)=0$ unless $1\le y\le M$.

Let

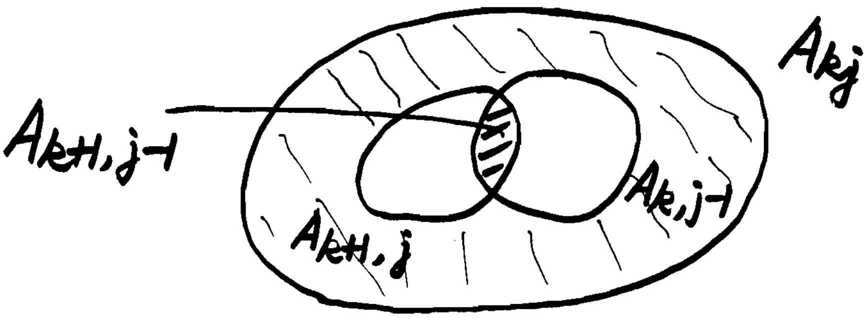
$$S_{kj}=\begin{cases}\sum_{y=k}^{j} f(y)=P[k\le Y\le j] & k\le j\\ 0 & \text{otherwise}\end{cases}$$

Compute and store for $1\le k\le j\le M$.

Then for $Y_1, Y_2, \cdots, Y_n$ iid with marginal dsn f. let

$$A_{kj} = \{ k \le \text{each } Y_i \le j \}$$

$$B_{kj} = \{ \min Y_i = k \text{ and } \max Y_i = j \}$$



$$B_{kj} = A_{kj} - A_{k+1,j} - A_{k,j-1} + A_{k+1,j-1}$$

Then $P(A_{kj}) = (S_{kj})^n$

$$M_{kj} = P(B_{kj}) = (S_{kj})^n - (S_{k+1,j})^n - (S_{k,j-1})^n + (S_{k+1,j-1})^n$$

Then

$$P(R=r) = \sum_{k=1}^{M-r} M_{k,k+r}$$

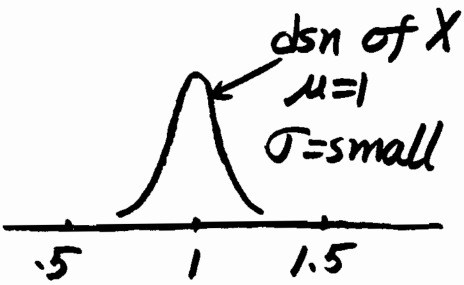
largest possible value of Y.

Discreteness/Crude Gaging and the Estimation of Parameters of an Underlying Conts Dsn

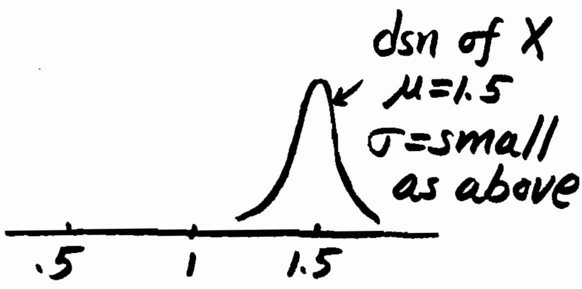
Suppose "really" X is normal (μ, σ^2) but we only observe X rounded to the nearest integer X^*.

You need to be very careful how you think here ... lots of things people naively expect don't hold true.

e.g., Suppose σ is small:



This will produce samples of X^*'s that are all 1's.



This will produce a sample of X^*'s that is a binomial mixture of 1's and 2's.

How to go from a sample of X^*'s to estimates for μ, σ? (sample mean and sample standard deviation may not be such a good way to estimate parameters)

Use the "likelihood function" idea.

— This is essentially to treat

$$P[\underbrace{\text{data}}_{x_1^*, x_2^*, \cdots, x_n^*} \mid \mu, \sigma] = L(\mu, \sigma)$$

as a (random) function of parameters guiding inference about parameters.

$$L(\mu,\sigma) = \prod_{i=1}^{n} \left[\Phi\left(\frac{x_i^* + .5 - \mu}{\sigma}\right) - \Phi\left(\frac{x_i^* - .5 - \mu}{\sigma}\right) \right]$$

Φ → cdf of Std. Normal