Inspection Error and Attributes Acceptance Sampling

% Defective Context

Perspective B

a probabilistic description of a single inspection

		Inspection Result G	Inspection Result D	
Actual condition	G	$(1-w_G)\cdot(1-p)$	$w_G\cdot(1-p)$	$(1-p)$
	D	$w_D\cdot p$	$(1-w_D)\cdot p$	p
		$1-p^*$	$p^*=w_G(1-p)+(1-w_D)\cdot p$	

It's p^* and not p that drives P_a.

So using (n, c) from perspective B

$$P_a(p, W_G, W_D) = \sum_{x=0}^{c} \binom{n}{x} p^{*x} (1-p^*)^{n-x}$$

and a similar kind of analysis says that from perspective A

$$P_a(p, W_G, W_D) = \sum_{x=0}^{n} \frac{\binom{Np}{x}\binom{N(1-p)}{n-x}}{\binom{N}{n}} \cdot P[\overbrace{U+V}^{\text{independent}}\leq c],$$

where $U \sim Bin(x, 1-W_D)$, $V \sim Bin(n-x, W_G)$

"Clearly" non-zero W_G and W_D "change" P_a (from no-inspection-error values).

- Increasing W_G decreases P_a.
- Increasing W_D increases P_a.

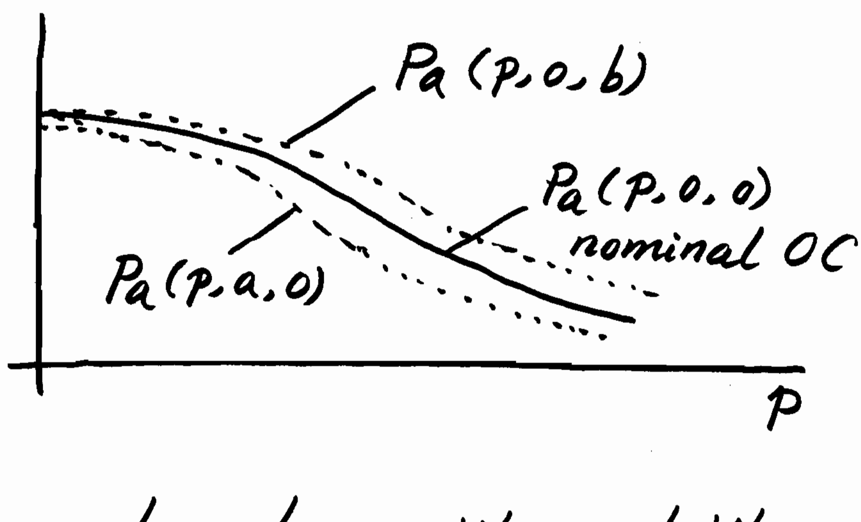
In fact, if I can bound

$$0 \le W_G \le a$$

$$0 \le W_D \le b$$

$$P_a(p, a, 0) \le \underset{P_a(p,0,0)}{\text{nominal } P_a} \le P_a(p, 0, b)$$

and I have



and my bounds on W_G and W_D guarantee that the real P_a is in that band.

A similar development can be done for <u>mean defects/unit</u>

Suppose that in an inspection real defects are each missed with probability m and that "phantom" defects are seen according to a Poisson process with rate λ_p (per unit).

Then for a given (n, c)

— from perspective B:

$$P_a(\lambda, m, \lambda_p) = P\left[\begin{matrix}\text{a Poisson r.v. with} \\ \text{mean } n(\lambda(1-m)+\lambda_p)\end{matrix} \le c\right]$$

— from perspective A, a reasonable acceptance probability is

$$P_a(T, m, \lambda_p) = P[U + V \le c]$$

$n\lambda$ $\text{Bin}\left(T, \left(\frac{n}{N}\right)(1-m)\right)$ independent $\text{Poisson}(n\lambda_p)$

General Attributes Acceptance Sampling

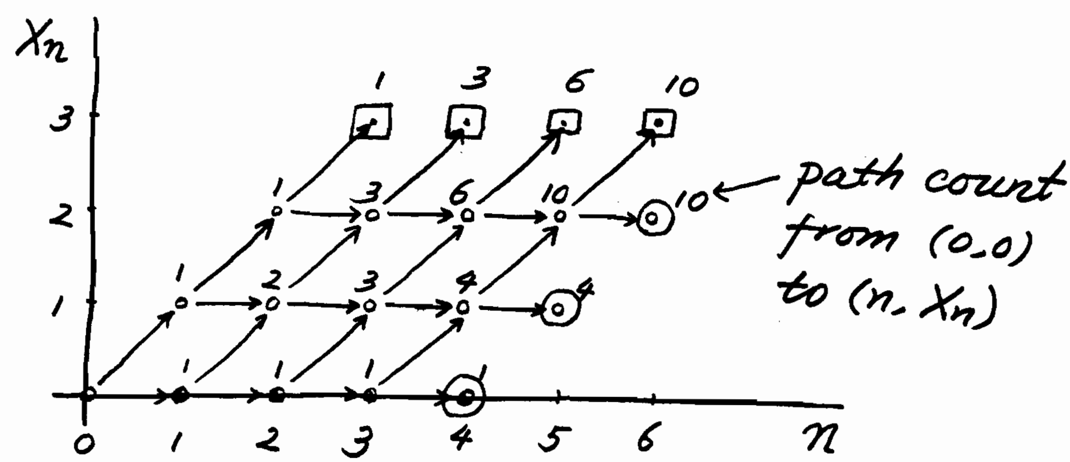
- The "all or none" result still holds if quality is fixed and known and costs are accrued on a per item basis.
- Bayesian/Economic choice of a general acceptance sampling is subtle:
  - It involves dynamic programming/backwards induction.
  - Not only can something other than "all" or "none" be optimal, but the overall best plan need not have fixed sample size.
- There are some "classical"/"two-point" design methods for certain types of plans more general than single sampling.

– For % defective contexts the statistical properties all possible are described very simply using the device of "path counting" on a diagram giving all possible evolutions of

$(n, \underbrace{X_n}_{})$. → # of defectives seen in n inspections

(The following substitutes for many lengthy and boring specialized developments in old SQC texts.)

<u>Example</u>. Doubly curtailed single sampling with $n=6$ and $c=2$.



□ = reject   o = accept

First consider type B calculations

$$P_a = 10 \cdot p^2 \cdot (1-p)^4 + 4p(1-p)^4 + 1 \cdot (1-p)^4$$

$$= \sum_{\substack{\text{acceptance} \\ \text{boundary}}} P[\text{ending at } (n, X_n)]$$

$$= \sum_{\substack{\text{acceptance} \\ \text{boundary}}} \begin{pmatrix} \text{path count} \\ \text{from } (0,0) \\ \text{to } (n, X_n) \end{pmatrix} \cdot p^{X_n} \cdot (1-p)^{n-X_n}$$

If $n = \#$ actually inspected, then

$En = ASN$ (Average Sample Number)

$$= \sum_{\substack{\text{stopping} \\ \text{boundary}}} n \cdot P[\text{ending at } (n, X_n)]$$

$$= 3 \cdot 1 \cdot p^3 + 4\left[(1-p)^4 + 3p^3(1-p)\right]$$

$$+ 5\left[4p(1-p)^4 + 6p^3(1-p)^2\right]$$

$$+ 6\left[10p^2(1-p)^4 + 10p^3(1-p)^3\right]$$

And in the context of "Rectifying Inspection" (100% inspection of rejected lots and replacement of all D's with G's in all inspections)

AOQ = average outgoing quality
= mean fraction defective exiting the 2-stage scheme

$$= \frac{1}{N} \sum_{\substack{\text{acceptance} \\ \text{boundary}}} (N-n)p \cdot P_r\left[\begin{array}{c}\text{ending at} \\ (n, X_n)\end{array}\right]$$

<u>Example</u> Suppose $N=10$.

$$AOQ = \left(1-\tfrac{4}{10}\right)p\cdot(1-p)^4 + \left(1-\tfrac{5}{10}\right)p\cdot 4p(1-p)^4$$
$$+ \left(1-\tfrac{6}{10}\right)p \cdot 10p^2(1-p)^4$$

and

ATI = mean # of items inspected per lot in this 2-stage scenario

$$= \sum_{\substack{\text{acceptance} \\ \text{boundary}}} n \cdot P_r[\text{ending at } (n, X_n)] + \sum_{\text{rejection boundary}} N \cdot P_r[\text{ending at } (n, X_n)]$$

$$= N(1-P_a) + \sum_{\substack{\text{acceptance} \\ \text{boundary}}} n \cdot P_r[\text{ending at } (n, X_n)]$$

Comments

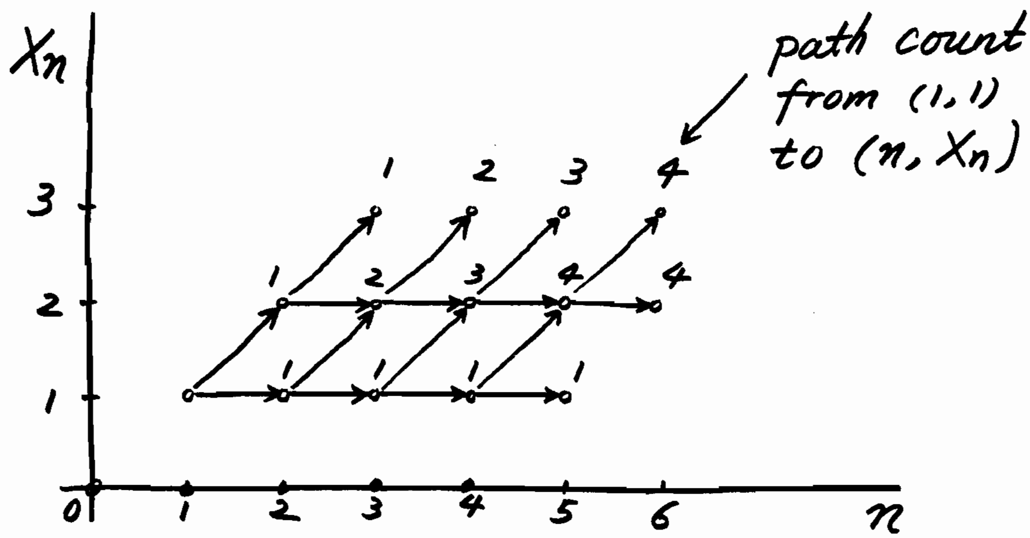
1) Type A calculations are just like type B calculations except

$$P_r[\text{ending at } (n, X_n)] = \begin{pmatrix} \text{path count} \\ \text{from } (0,0) \\ \text{to } (n, X_n) \end{pmatrix} \cdot \frac{\binom{N-n}{N\cdot p - X_n}}{\binom{N}{N\cdot p}}$$

2) For both type A and type B contexts it's true that

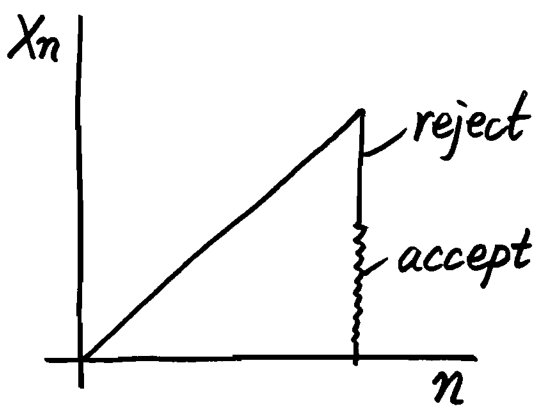
$$\text{UMVUE of } p = \frac{\text{path count from } (1,1) \text{ to } (n, X_n)}{\text{path count from } (0,0) \text{ to } (n, X_n)}$$

Example. Doubly curtailed single sampling with $n=6$, $c=2$.

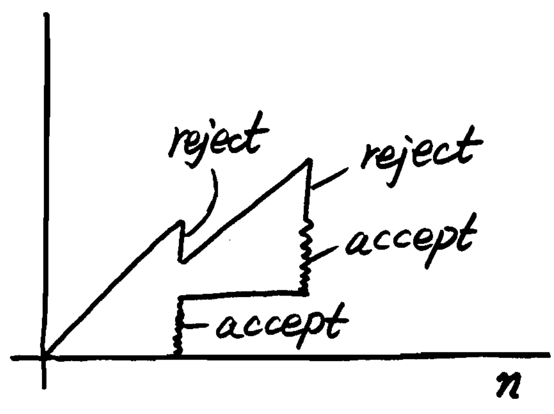


Stop sampling point (n, X_n)	$\tilde{p}$ = UMVUE	$\hat{p} = \frac{X_n}{n}$ type B MLE
$(3, 3)$	$1/1 = 1$	1
$(4, 0)$	$0/1 = 0$	0
$(4, 3)$	$2/3$	$3/4$
$(5, 1)$	$1/4$	$1/5$
$(5, 3)$	$3/6$	$3/5$
$(6, 2)$	$4/10$	$2/6$
$(6, 3)$	$4/10$	$3/6$

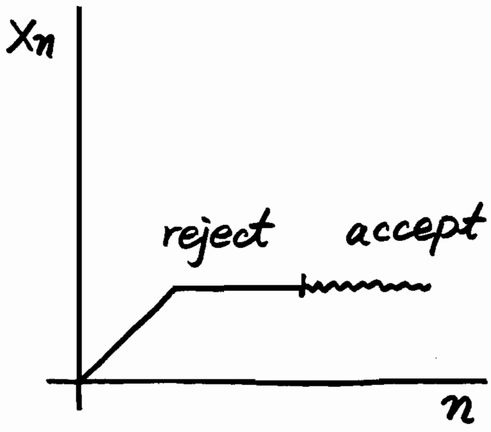
3) Various kinds of "standard" % defective plans simply have different shaped acceptance and rejection boundaries.



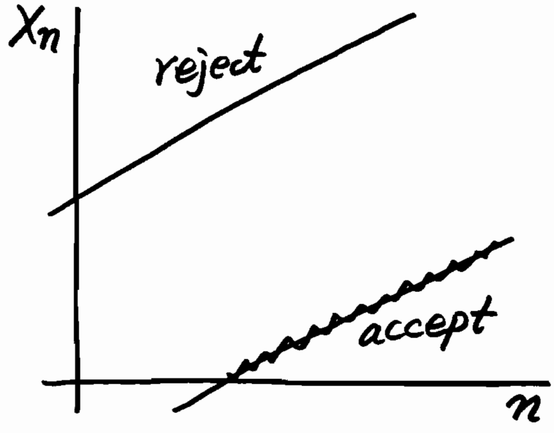
usual single sampling



double sampling



inverse sampling



"sequential" sampling
SPRT (sequential probability ratio test)

"Variables" Acceptance Sampling (see §8.2 of V&J)

X: some cont$^{\underline{s}}$ measurement
(used to define G/D)

$X \sim f(x|\theta)$ $\quad\quad \theta \in \Theta$
possibly k-dimensional

Specifications $\quad\quad L$ and/or U

Let

$$p(\theta) = \begin{cases} \int_U^\infty f(x|\theta)\,dx & \text{one-sided (upper) spec.s} \\ \int_{-\infty}^L f(x|\theta)\,dx & \text{one-sided (lower) spec.s} \\ \text{sum of above} & \text{two-sided case} \end{cases}$$

$X_1, X_2, \cdots, X_n$ iid $f(x|\theta)$

$\phi(\underset{\sim}{x})$: a decision procedure

(i.e., $\phi(\underset{\sim}{x}) = 0$ or 1.

meaning: 1 = reject lot and

0 = accept lot)

We'd like:

1) $\phi(\underset{\sim}{X})$ to tend to take the value 1 when $p(\theta)$ is large and tend to take the value 0 when $p(\theta)$ is small.

and

2) if θ and θ' are elements of Θ so that $p(\theta) = p(\theta')$, then

$P_a(\theta) \equiv P_\theta[\phi(\underset{\sim}{X})=0] \approx P_{\theta'}[\phi(\underset{\sim}{X})=0] \equiv P_a(\theta')$.

Read §8.2 for some critique of these objectives. It's not clear that they are really rational. Another issue is that measurement error really wrecks havoc with these goals.

The most famous version is the normal / Gaussian version

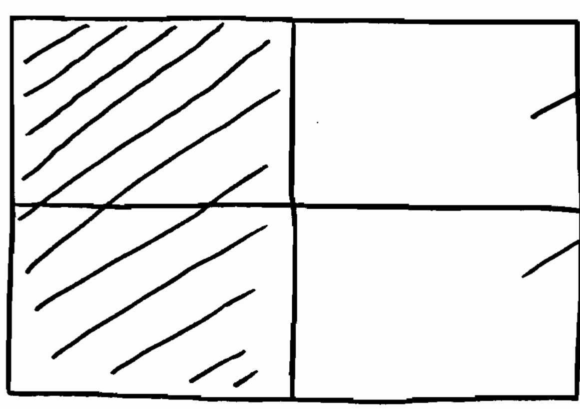
Model: $X_1, X_2, \cdots, X_n$ iid $N(\mu, \sigma^2)$

This is sensible if

A (1) $N >> n$. simple random sampling and lot relative frequency dsn is approximately Gaussian

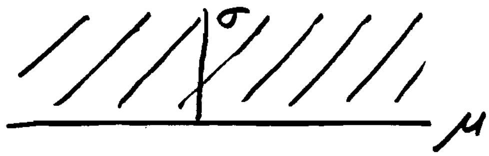
B (or 2) iid Gaussian stable process model describing generation of items in lot (and I'm really doing inference on that process).

Standard Methods / Scenarios

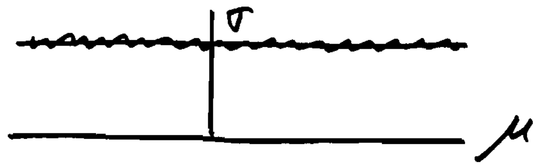


What does "known σ" mean?

Instead of parameter space



I have instead



Therefore 1) I may use σ in my prescription for $\phi(\underset{\sim}{X})$

2) I will only evaluate the performance of $\phi(\underset{\sim}{X})$ on this single line in (μ, σ)-plane.