We'll work through parts a) & b) of Problem 3.6 of the notes:

$$\varepsilon(-1),\ \varepsilon(0),\ \varepsilon(1),\ \cdots \quad \text{iid } N(0,\sigma^2)$$

$$Z(t) = \varepsilon(t-1) + \varepsilon(t), \quad t = 0, 1, 2, \cdots$$

$$\begin{bmatrix} Z(0) \\ Z(1) \\ Z(2) \\ \vdots \\ Z(k) \end{bmatrix} \sim MVN_{k+1}(\underset{\sim}{0}, \underset{\sim}{\Sigma}_k)$$

where

$$\underset{\sim}{\Sigma}_k = \begin{bmatrix} 2\sigma^2 & \sigma^2 & 0 & & & & \\ \sigma^2 & 2\sigma^2 & \sigma^2 & 0 & & 0 & \\ & \sigma^2 & 2\sigma^2 & \sigma^2 & 0 & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & 0 & & \sigma^2 & 2\sigma^2 & \sigma^2 \\ & & & & \sigma^2 & 2\sigma^2 \end{bmatrix}$$

Suppose $T(t) = 0 \quad \forall t$ (for all t)

and first a) $A(a,s) = a \quad \forall s \geq 1$

then b) $A(a,s) = \begin{cases} 0 & s = 1 \\ a & \forall s \geq 2 \end{cases}$

MV Control Policies?

<u>Facts</u> (from facts about MVN dsns)

$$\hat{z}(s|t) = ?$$

$$\hat{z}(1|0) = \tfrac{1}{2} z(0)$$

$$\hat{z}(2|1) = \tfrac{2}{3} z(1) - \tfrac{1}{3} z(0)$$

$$\hat{z}(3|2) = \tfrac{3}{4} z(2) - \tfrac{2}{4} z(1) + \tfrac{1}{4} z(0)$$

$$\hat{z}(4|3) = \tfrac{4}{5} z(3) - \tfrac{3}{5} z(2) + \tfrac{2}{5} z(1) - \tfrac{1}{5} z(0)$$

$$\hat{z}(t+1|t) = \frac{1}{t+2} \sum_{j=0}^{t} (-1)^j (t+1-j) z(t-j)$$

for $s > t+1$ $\quad \hat{z}(s|t) = 0.$

<u>Case a)</u>

<u>at time $t=0$</u> $\quad \hat{z}(1|0) = \tfrac{1}{2} z(0)$

So choose $a(0)$ so that

$$A(a(0), 1) = a(0) = -\tfrac{1}{2} z(0)$$

<u>at time $t=1$</u>

$$z(1) = Y(1) - A(a(0), 1)$$

$$= Y(1) - \left(-\tfrac{1}{2}Z(0)\right)$$

$$= Y(1) + \tfrac{1}{2}Z(0)$$

$$\hat{Z}(2|1) = \tfrac{2}{3}Z(1) - \tfrac{1}{3}Z(0) = \tfrac{2}{3}Y(1)$$

$$\hat{Y}(2|1) = \tfrac{2}{3}Y(1) + \Big(\underbrace{-\tfrac{1}{2}Z(0)}_{A(a(0),2)=a(0)} + \underbrace{A(a(1),1)}_{a(1)}\Big)$$

Since $T(t) = 0 \quad \forall\, t$,

set $\quad \tfrac{2}{3}Y(1) + \left(-\tfrac{1}{2}Z(0) + a(1)\right) = 0$

and solve for $a(1)$, i.e., take

$$a(1) = -\tfrac{2}{3}Y(1) + \tfrac{1}{2}Z(0)$$

<u>at time $t=2$</u>

$$Z(2) = Y(2) - \Big[\overbrace{-\tfrac{2}{3}Y(1) + \tfrac{1}{2}Z(0)}^{\substack{A(a(1),1)\\ \shortparallel \\ a(1)}} + \overbrace{\left(-\tfrac{1}{2}Z(0)\right)}^{\substack{A(a(0),2)\\ \shortparallel \\ a(0)}}\Big]$$

$$= Y(2) + \tfrac{2}{3}Y(1)$$

$$\hat{Z}(3|2) = \tfrac{3}{4}Z(2) - \tfrac{2}{4}Z(1) + \tfrac{1}{4}Z(0)$$

$$= \tfrac{3}{4}Y(2)$$

Choose $a(2)$ so that

$$\underbrace{A(a(2),1)}_{a(2)} = \underbrace{-\hat{Y}(3|2)}$$

$$= -\left[\hat{Z}(3|2) + A(a(0),3) + A(a(1),2)\right]$$

Thus $a(2) = -\left[\frac{3}{4}Y(2) - \frac{2}{3}Y(1)\right]$.

In general

$$a(t) = -\left[\frac{t+1}{t+2}Y(t) - \frac{t}{t+1}Y(t-1)\right]$$

as $t \to \infty$, $a(t) \approx -Y(t) + Y(t-1)$

i.e., $a(t) \approx E(t) - E(t-1)$

$= \Delta E(t)$

i.e., here the MV controller is a proportional-only algorithm.

Case b: at time $t=0$ the earliest Y we can affect by choice of $a(0)$ is $Y(2)$.

Thus $\hat{Z}(2|0) = 0 = T(2)$.

i.e., we "expect" to be on target at time 2 without adjustment.

— So take $a(0) = 0$.

Similarly $a(t) = 0 \quad \forall t \geq 0$.

Chapter 5 of V&J

Process Characterization and "Capability" Analysis

5.1 Graphical Tools

5.2 Process Capability Measures and Estimation

(stability here is iid-like behavior)

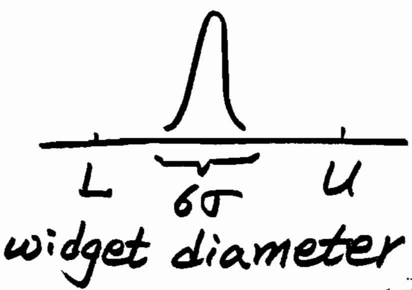
(<u>normal</u> process)

- 6σ "process capability"

Usual confidence limits (based on $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$) for σ give confidence limits for 6σ.

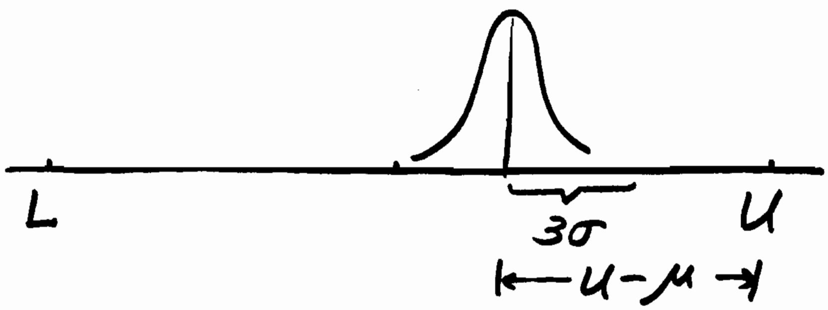
- $C_p = \frac{U-L}{6\sigma}$ "process capability ratio"

Usual confidence limits for σ give confidence limits for C_p.



- $C_{pk} = \min\left\{ \frac{U-\mu}{3\sigma}, \frac{\mu-L}{3\sigma} \right\}$

  — the 'number of '3-sigma' that the mean is to the good side of the nearest specification'



Inference? / Confidence Limits?

$$\hat{C}_{pk} = \min\left\{ \frac{U-\bar{x}}{3s}, \frac{\bar{x}-L}{3s} \right\}$$

Confidence limit (lower confidence bound):

$$\hat{C}_{pk} - z \cdot \underbrace{\sqrt{\frac{1}{9n} + \frac{\hat{C}_{pk}^2}{2(n-1)}}}_{\Delta \text{ method standard error}}$$