Stat/IE 531 **Midterm Exam** **March 27, 2007**

Points for each question are indicated in the left margin in square brackets.

[20] 1. In a calibration study, $n = 40$ observations were obtained on load cell calibration, with x being the load, and y being the deflection. Fitting a simple regression to the data gives the following fitted equation

$$y = 0.00615 + 0.00722x.$$

We also have that

$$\overline{x} = 157.5, \quad \sum_{i=1}^{40}(x_i - \overline{x})^2 = 299250,$$

and the root mean squared error

$$\sqrt{\text{MSE}} = 0.002171.$$

[6] (a) The deflection of a new specimen is measured as 1.200. Give a point estimate for the true load of this specimen.

[14] (b) Give an approximate 90% confidence interval for the true load of the specimen in part (a).

[30] 2. Consider the standards given c chart based on a stable Poisson process model (i.e., the counts of defects on each unit are iid with a Poisson distribution).

[6] (a) Give the control limits for $\lambda = 1$.

[6] (b) Find the average run length (ARL) of the c chart in part (a) when $\lambda = 1$ (all-OK ARL).

[6] (c) To achieve an all-OK ARL of at least 500, what control limits should be used in part (a)?

[6] (d) Find the ARL of the c chart in part (a) when $\lambda = 2$.

[6] (e) Apply your c chart in part (a) to the following counts of defects on 10 inspected units:

$$0, 2, 2, 1, 4, 5, 3, 1, 6, 3$$

(Plot the observations on the chart.) Are any alarms signaled?

[25] 3. A company that makes rubber hoses is interested in monitoring the lengths of the hoses. Assume that under stable conditions the lengths are normally distributed with $\mu = 100$ inches and $\sigma = 0.2$ inch.

[10] (a) Suppose that quickest possible detection of a 0.3 inch change in the mean length (from the target value of 100 inches) is desired. Design a two-sided CUSUM scheme for $\bar{x}$'s based on samples of size 5, using the above μ and σ and an all-OK ARL of 500. (Give reference values k_1 and k_2, starting values u and v, and a decision interval h.)

[15] (b) The company is currently using a two-sided CUSUM scheme with $k_1 = 100.2, k_2 = 99.8, u = v = 0, h_1 = h_2 = 0.3$, and $n = 4$. Apply this scheme to the data below. (Plot appropriate quantities on the CUSUM scheme.) Are any alarms signaled?

Subgroup	1	2	3	4	5	6
$\bar{x}$	100.2	99.9	99.8	100.6	99.6	99.5

[25] 4. Consider monitoring nonnegative integer-valued (i.e., taking values of $0, 1, 2$, etc.) random variables $Q_1, Q_2, Q_3, \ldots$ with the following alarm rules:

(1) alarm at time $i = 1$ if $Q_1 \geq 3$, and

(2) for $i \geq 2$ alarm at time i if $Q_{i-1} + Q_i \geq 3$.

For integer $j \geq 0$, let $q_j = P[Q_1 = j]$ and suppose that the Q_i's are iid. Set up a Markov Chain that you can use to find the ARL of this scheme under the iid model for the Q's. (Be sure to carefully and completely define your state space, write out the proper transition matrix, and be explicit about how you would obtain the desired ARL.)