Stat/IE 531 **Final Exam** **May 4, 2007**

Please turn in both your questions and answers. They will be returned to you later.

Points for each question are indicated in the left margin in square brackets.

[24] 1. A company that makes rubber hoses is interested in monitoring the lengths of the hoses. Assume that under stable conditions the lengths are normally distributed with $\mu = 100$ inches and $\sigma = 0.2$ inch.

[8] (a) Suppose that quickest possible detection of a 0.3 inch change in the mean length (from the target value of 100 inches) is desired. Design a two-sided EWMA scheme for $\overline{x}$'s based on samples of size 5, using the above μ and σ and an all-OK ARL (average run length) of 370.

[8] (b) If the process mean is actually 101 and σ is 0.3, obtain the ARL of your scheme in part (a).

[8] (c) The company is currently using a two-sided EWMA scheme with $UCL_{EWMA} = 100.25$ and $LCL_{EWMA} = 99.75$. Begin with a starting value of $EWMA_0 = 100$ and $\lambda = 0.3$, and apply this scheme to the data below. (No need to give an EWMA chart.) Are any alarms signaled?

Subgroup	1	2	3	4	5	6
$\overline{x}$	100.2	99.9	99.8	100.6	99.6	99.5

[18] 2. Suppose that the use of the I controller $\Delta X(t) = E(t)$ turns out to be the best controller for the process $Z(t)$, $t \geq 0$; that is, the observed (controlled) process $Y(t)$ $(t \geq 0)$ are independent and identically distributed (iid) $N(0, \sigma^2)$ random variables. Suppose that

(1) the target for the controlled variable is $T(t) = 0$ for $t \geq 0$

(2) control begins at time 0 (after observing $Z(0)$)

(3) the effect of a control action on $Y(s)$ is $\Delta X(t)$ for all $s \geq t + 1$

[8] (a) Show that $Z(t)$ can be written as

$$Z(t) = \sum_{s=0}^{t} e(s), \quad \text{for } t \geq 0,$$

for some iid $N(0, \sigma^2)$ random variables $e(s)$ $(s \geq 0)$.

[10] (b) If an I controller $\Delta X(t) = 0.5E(t)$ $(t \geq 0)$ were used instead, it would no longer be the best controller. With the above assumptions (1) to (3) and $Z(t)$ given in part (a), show that the observed process $Y(t)$ now follows the following model:

$$Y(t) - 0.5Y(t-1) = e(t), \quad t \geq 1,$$

where $e(t)$ $(t \geq 1)$ are iid $N(0, \sigma^2)$ random variables.

[16] 3. The following are diameter measurements taken on $n = 5$ cylindrical holes. The units are inches.

$$9.9983,\ 9.9960,\ 10.0028,\ 10.0042,\ 9.9967$$

The data give $\overline{x} = 9.9996$ and $s = 0.00369$. The specifications for this diameter measurement were 10 ± 0.005 inches. Assume that the process producing these diameters is stable and that the diameters follow a normal distribution.

[8] (a) Give a 95% lower confidence bound for the process capability ratio C_{pk}.

[8] (b) Give a 95% upper statistical tolerance bound for 99% of all diameter measurements of cylindrical holes produced by this process.

[24] 4. Consider the following percent defective acceptance sampling plan. For

$$X_n = \text{the number of defective items found through the } n\text{th item inspected,}$$

the plan rejects the lot if $X_8 \geq 3$ and accepts the lot if $X_8 \leq 2$, or as soon as such conditions can be determined.

[8] (a) Find expressions for the OC (operating characteristic) and the ASN (average sample number) of this plan.

[8] (b) Find expressions for the AOQ (average outgoing quality) and ATI (average number of items inspected per lot) of this plan, if it is used in a rectifying inspection scheme for lots of size $N = 100$.

[8] (c) What is the UMVUE (uniformly minimum variance unbiased estimator) of p (the percent defective) of this plan? Say what value one should estimate for every possible stop-sampling point.

[18] 5. Consider the Deming inspection scenario with $N = 10, k_1 = 1, k_2 = 12$, and a prior distribution G for p with $P[p = 0] = P[p = .1] = P[p = .2] = 1/3$.

[12] (a) For each sample size $n = 0, 1, 2$, find the best fixed n inspection plan that minimizes the expected total costs. Be specific about when you would reject or accept the lot.

[6] (b) Which sample size ($n = 0, 1, 2$) is best here? (Show calculations to support your answers.)