

Points for each question are indicated in the left margin in square brackets.

- [24] 1. In a Gage R&R study involving 10 parts and 2 operators, each operator took 3 measurements from each part. A 2-way ANOVA of the resulting data gave the mean squares associated with the part, operator, part  $\times$  operator, and measurement error as 10, 12, 6, and 2, respectively.
- [10] (a) Give point estimates of the reproducibility and repeatability standard deviations.
- [14] (b) Give standard errors (estimated standard deviations) for the estimates in part (a).
- [36] 2. Two students did a project with a company that cuts rolled paper into sheets. Data below summarize 15 sampled subgroups of  $n = 5$  consecutive sheets as they were cut. (The measurements were in 64ths of an inch above nominal sheet length.)

Subgroup	$\bar{x}$	$s$
1	12.2	.84
2	11.2	1.64
3	10.6	2.07
4	12.2	2.49
5	11.2	.84
6	12.6	1.82
7	12.2	2.95
8	13.6	1.67
9	12.2	1.30
10	10.4	1.52
11	10.4	1.95
12	10.6	1.67
13	10.4	1.67
14	12.0	2.91
15	11.2	.84
	173.0	26.18

- [7] (a) Find retrospective control limits for the subgroup means.
- [7] (b) Find retrospective control limits for the subgroup standard deviations.
- [7] (c) What do the charts in parts (a) and (b) indicate (in retrospect) about the stability of the cutting process?

- [15] (d) Suppose that the process is stable and sheet length is normally distributed with mean 10 (in 64ths of an inch above nominal) and standard deviation 1.5. Consider monitoring this process based on samples of size  $n = 5$  using the control charts of (a) and (b).
- (i) Calculate the ARL of the chart in part (a).
  - (ii) Calculate the ARL of the chart in part (b). (You may express your result using the cumulative distribution function of a known distribution.)
  - (iii) Calculate the ARL of the combined chart using the control limits in parts (a) and (b). (You may express your result using the cumulative distribution function of a known distribution.)
- [20] 3. Consider monitoring a process that produces normally distributed observations with  $\sigma = 10$  under stable conditions.
- [10] (a) Give reference values  $k_1$  and  $k_2$ , sample size  $n$ , starting values  $u$  and  $v$ , and a decision interval  $h$  so that a two-sided CUSUM scheme for  $\bar{x}$ 's for monitoring the process mean  $\mu$  will have an ARL (average run length) of 370 if  $\mu = 100$  and an ARL of 2.5 if  $\mu = 108$ .
  - [10] (b) Obtain the ARL of your CUSUM scheme in part (a) if in fact the process mean is 110 and the process standard deviation is 9.
- [20] 4. Consider a Shewhart control chart with the following two alarm rules:
- signal if a single point falls outside 3 sigma control limits
  - signal if 2 out of 3 consecutive points are outside 2 sigma limits

Suppose that points  $Q_1, Q_2, Q_3, \dots$  are to be plotted on this chart and that the  $Q$ 's are iid.

Set up a Markov Chain that you can use to find the ARL of this scheme under the iid model for the  $Q$ 's. (Be sure to carefully and completely define your state space, write out the proper transition matrix and be explicit about how you would obtain the desired ARL.)