Please turn in both your questions and answers. They will be returned to you later.

Points for each question are indicated in the left margin in square brackets.

[20] 1. Consider monitoring a process based on samples of size $n = 9$. Suppose that the process produces normally distributed observations with $\mu = 200$ and $\sigma = 1.5$ under stable conditions.

[12] (a) Suppose that an all-OK ARL (average run length) of around 100 and quickest possible detection of the process mean off target by 1 are desirable. Set up an EWMA chart for this situation. Does a value of $\overline{x} = 201$ trigger an alarm?

[8] (b) If the process mean is actually 201 and σ is 2, obtain the ARL of your scheme in part (a).

[22] 2. Consider the use of the PI(D) controller $\Delta X(t) = 0.6\Delta E(t) + 0.3E(t)$. Suppose that

- the uncontrolled process $Z(t) = 1 + t$ for all $t \geq -1$
- the target for the controlled variable is $T(t) = 2$ for $t \leq 2$ and $T(t) = 5$ for $t > 2$
- control begins at time 0 (after observing $Z(0)$)
- the effect of a control action on $Y(s)$ is $\Delta X(t)$ for all $s \geq t + 1$ (i.e., $Y(t) = Z(t) + \sum_{s=0}^{t-1} \Delta X(s)$).

[12] (a) Find the values of $Z(t), Y(t)$, and $E(t)$ for $t = 0, 1, 2, 3, 4$, and 5.

[10] (b) Find the best controller $\Delta X(t)$ (not necessarily a PID controller) so that $Y(t) = Z(t) + \sum_{s=0}^{t-1} \Delta X(s)$ is equal to the target $T(t)$ for all $t \geq 1$.

[15] 3. In an experiment made on the amount of carbon in cast iron, 2 heats of cast iron were studied on each of 2 days, and 3 determinations of carbon content were made for each of the first three heats and 4 determinations were made for the last heat. Assume that the resulting data, y_{ijk}'s, can be described by a hierarchical normal random effects model. Find sensible point estimates of the parameters σ_α^2, σ_β^2, and σ^2 (the "day", "heat(day)", and "error" variance components).

[25] 4. Consider the following percent defective acceptance sampling plan. For

$$X_n = \text{the number of defective items found through the } n\text{th item inspected,}$$

the plan rejects the lot if $X_n \geq \min(3, 1 + 0.5n)$ and accepts the lot if $X_n \leq 0.4n - 1$ or after inspecting a maximum of 7 items without rejecting the lot.

[9] (a) Find expressions for the OC (operating characteristic) and the ASN (average sample number) of this plan.

[8] (b) Find expressions for the AOQ (average outgoing quality) and ATI (average number of items inspected per lot) of this plan, if it is used in a rectifying inspection scheme for lots of size $N = 100$.

[8] (c) What is the UMVUE (uniformly minimum variance unbiased estimator) of p (the percent defective) of this plan? Say what value one should estimate for every possible stop-sampling point.

[18] 5. Consider single sampling based on a normally distributed variable.

[10] (a) Find a double limits variables sampling plan with $L = 1.14$, $U = 1.16$, $\sigma = 0.003$, $p_1 = 0.05$, $Pa_1 = 0.9$, $p_2 = 0.1$, and $Pa_2 = 0.1$.

[8] (b) How does n in part (a) compare with what would be required for an attributes sampling plan with a comparable OC curve?