

Student Name:

Ground Rules:

- Open Book, Open Notes (You may use any references you like.)
- Calculator Allowed
- 2 Hour Time Limit
- Write on Questions 1, 2, 3, and one of 4 and 5.
- If you write on both #4 and #5, I will score your better answer of the two.
- Please turn in both your questions and answers. They will be returned to you later.

[April 3 (Target Date) for Off-Campus Students]

Do problems 1 through 3 and then **one of problems 4 and 5**. Points for each question are indicated in the left margin in square brackets.

- [25] 1. Two samples of size $n = 5$ body-bound bolt shanks were taken from a production run and their diameters were measured to the nearest .0001 inch, producing the following data. Diameters produced in each production run are believed to be approximately normally distributed.

Sample	1	2
Measurements	.3473, .3473, .3473, .3473, .3473	.3473, .3474, .3474, .3473, .3474

- [10] (a) Use sample #1 only and give approximate 95% confidence limits for (1) the mean diameter and (2) the standard deviation of diameters for this production run.
- [15] (b) Use both samples and give approximate 95% confidence limits for (1) the mean diameter and (2) the standard deviation of diameters for this production run.
- [25] 2. Consider the monitoring of a process based on samples of size $n = 4$. Assume that under stable conditions the process produces normally distributed observations with $\sigma = .04$.
- [9] (a) Set up a high side CUSUM scheme for \bar{x} 's for monitoring the process mean, using a target value of 10 and an all-OK ARL (average run length) of 370. Quickest possible detection of a .04 increase in the mean (from the target) is desired. (Give values of k_1 , u_0 and h_1 .)
- [8] (b) Obtain the ARL of your CUSUM scheme in part (a) if in fact the process mean is 10.03 and the process standard deviation remains at .04.
- [8] (c) For a Shewhart \bar{x} chart with an all-OK ARL of 370, find its ARL if in fact the process mean is 10.03 and the process standard deviation remains at .04.

- [25] 3. A manufacturer of U-bolts for the auto industry measures and records thread lengths on bolts that it produces. Below are some summary statistics taken from 18 subgroups, with each subgroup containing $n = 5$ consecutive bolts. (The units are .001 inch above nominal.)

Subgroup	s	\bar{x}	R
1	2.61	11.4	6
2	1.64	11.2	4
3	2.51	11.6	6
4	2.05	11.2	5
5	1.10	11.2	3
6	1.22	11.0	3
7	2.30	9.4	5
8	1.73	11.0	4
9	1.52	9.6	3
10	3.21	9.4	7
11	2.30	11.4	6
12	2.17	10.8	6
13	2.05	11.2	5
14	1.34	10.4	3
15	.89	11.4	2
16	.55	13.4	1
17	1.22	13.0	3
18	1.10	11.8	2
	31.51	200.4	74

- [6] (a) Estimate the supposedly common subgroup standard deviation, σ , using (1) the subgroup ranges (R_i) and (2) the subgroup standard deviations (s_i).
- [7] (b) Find retrospective control limits for the subgroup standard deviations. Use your estimate of σ based on the R_i .
- [6] (c) Find retrospective control limits for the subgroup means. Use your estimate of σ based on the R_i .
- [6] (d) What does the chart in part (c) suggest about the threading process?

- [25] 4. The following is an ANOVA table from a real Gage R&R study for paper burst strength involving 2 pieces of paper, 5 operators and 2 measurements for each piece \times operator combination.

ANOVA Table			
Source	<i>SS</i>	<i>df</i>	<i>MS</i>
Piece	.5445	1	.5445
Operator	2.6920	4	.6730
Piece \times Operator	24.4980	4	6.1245
Error	20.9550	10	2.0955
Total	48.6895	19	

- [10] (a) Give a point estimate of the standard deviation of burst measurements on a fixed piece of paper made by many operators.
- [15] (b) Give a standard error (an estimated standard deviation) for the estimate in part (a). (Plug in numbers, but you don't need to simplify.)

- [25] 5. Consider a Shewhart control chart with the following two alarm rules:

- signal if a single point falls outside 3 sigma control limits
- signal if 4 consecutive points are outside 1 sigma limits

Suppose that points Q_1, Q_2, Q_3, \dots are to be plotted on this chart and that the Q 's are iid.

Set up a Markov Chain that you can use to find the ARL of this scheme under the iid model for the Q 's. (Be sure to carefully and completely define your state space, write out the proper transition matrix and be explicit about how you would obtain the desired ARL.)