Student Name:

Ground Rules:

- Open Book, Open Notes (You may use any references you like.)
- Calculator Allowed
- 2 Hour Time Limit
- Please turn in both your questions and answers. They will be returned to you later.

[May 16 (Target Date) for Off-Campus Students]

Points for each question are indicated in the left margin in square brackets.

[15] 1. Consider the monitoring of a process based on samples of size $n = 4$. Assume that under stable conditions the process produces normally distributed observations with $\sigma = .2$.

[8] (a) Design an EWMA scheme for $\overline{x}$'s for monitoring the process mean, using a target value of 5 and an all-OK ARL (average run length) of 500. Quickest possible detection of a change in the process mean of magnitude .2 is desired.

[7] (b) If the process mean is really 5.3 and σ is .3, obtain the ARL of your scheme in part (a).

[25] 2. Consider a stochastic control problem with the following uncontrolled process model $\mathcal{F}$:

$$Z(t) = \alpha + \beta t + \epsilon(t),$$

where α and β are known constants and the $\epsilon(t)$'s are iid normal $(0, \sigma^2)$ random variables. Suppose that the effect of a control action a taken s periods previous can be described as $A(a, s) = (1 - 2^{-s})a$ for all $s \geq 1$. Suppose further that the target value $T(t) = 0$ for all t and that control begins at time 0 (after observing $Z(0)$).

[5] (a) Find $\widehat{Z}(s|t) = E_{\mathcal{F}}(Z(s)|\ldots, Z(-1), Z(0), \ldots, Z(t))$, for $s > t$.

[6] (b) Find the optimal control actions $a(0), a(1)$, and $a(2)$ as functions of $Z(0), Y(1)$, and $Y(2)$.

[8] (c) Find the optimal control actions $a(t)$ for $t \geq 3$ (i.e., give the general minimum variance control algorithm). Does there seem to be a limiting form for $a(t)$? Explain.

[6] (d) Find $Y(t)$ for $t \geq 1$. Then use your result to show that minimum variance control will improve process performance.

[20] 3. In an experiment made on the amount of carbon in cast iron, 2 heats of cast iron were studied on each of 2 days and 3 determinations of carbon content were made for each of these heats, producing the following ANOVA table. Assume that the data can be described by a hierarchical normal random effects model.

Source	SS	df	MS
Day	.0184	1	.0184
Heat(day)	.0027	2	.0014
Error	.2044	8	.0256
Total	.2255	11	

[7] (a) Find point estimates of the parameters σ_α^2, σ_β^2, and σ^2 (the "day", "heat(day)", and "error" variance components).

[7] (b) Find a standard error for your estimate of σ_α^2 in part (a).

[6] (c) If 10 heats of cast iron were made each day for the next 5 days, and 1 determination of carbon content was made for each of these heats, give a numerical prediction of the overall sample variance of these future 50 measurements based on your estimates in part (a).

[25] 4. Consider the following percent defective acceptance sampling plan. For

$$X_n = \text{the number of defective items found through the } n\text{th item inspected,}$$

the plan rejects the lot if $X_n \geq 2$ and $X_n \geq .5n$, and accepts the lot after inspecting a maximum of 8 items without rejecting the lot.

[9] (a) Find expressions for the OC (operating characteristic) and the ASN (average sample number) of this plan.

[8] (b) Find expressions for the AOQ (average outgoing quality) and ATI (average number of items inspected per lot) of this plan, if it is used in a rectifying inspection scheme for lots of size $N = 50$.

[8] (c) What is the UMVUE (uniformly minimum variance unbiased estimator) of p (the percent defective) of this plan? Say what value one should estimate for every possible stop-sampling point.

[15] 5. Consider the Deming inspection scenario with $N = 2, k_1 = 1, k_2 = 5$ and a prior distribution G for p with $P[p = .15] = P[p = .2] = P[p = .25] = 1/3$. Find the optimal fixed n inspection plan (n, c) that minimizes the expected total costs.