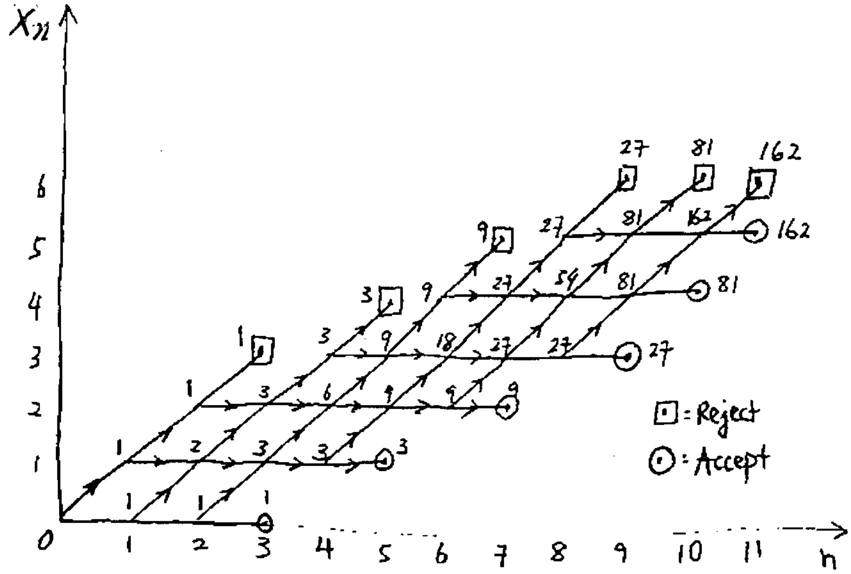
5.4



n	upper limit $1.5+0.5n$	lower limit $-1.5+0.5n$
1	2	-1
2	-2.5	-0.5
3	3	0
4	3.5	0.5
5	4	1
6	4.5	1.5
7	5	2
8	5.5	2.5
9	6	3
10	6.5	3.5
11	6	5

The table showing the acceptance & rejection boundaries

reject if $X_n \geq 1.5 + .5n$

accept if $X_n \leq -1.5 + .5n$

$A = \{(3,0), (5,1), (7,2), (9,3), (10,4), (11,5)\}$

(a). $OC = P_a(p) = \sum_{(n,X_n)\in A} \begin{pmatrix}\text{Path count from} \\ (0,0) \text{ to } (n,X_n)\end{pmatrix} p^{X_n}(1-p)^{n-X_n}$

$$= 1\cdot p^0(1-p)^3 + 3p(1-p^4) + 9p^2(1-p)^5 + 27p^3(1-p)^6 + 81p^4(1-p)^6 + 162p^5(1-p)^6$$

$$ASN = \sum_{(n,X_n)\in A\cup R} n \cdot Pr(\text{ending at } (n,X_n))$$

$$= 3\cdot\{p^0(1-p)^3 + p^3(1-p)^0\} + 5\cdot 3\{p(1-p)^4 + p^4(1-p)\} + 7\cdot 9\{p^2(1-p)^5 + p^5(1-p)^2\}$$

$$+ 9\cdot 27\cdot\{p^3(1-p)^6 + p^6(1-p)^3\} + 10\cdot 81\cdot\{p^4(1-p)^6 + p^6(1-p)^4\} + 11\cdot 162\{p^5(1-p)^6 + p^6(1-p)^5\}$$

(b). $N = 100$

$$AOQ = \sum_{(n,X_n)\in A}\left(1-\frac{n}{N}\right)p \cdot Pr(\text{ending at } (n,X_n))$$

$$= \left(1-\tfrac{3}{100}\right)p\cdot p^0(1-p)^3 + \left(1-\tfrac{5}{100}\right)p\cdot 3p(1-p)^4 + \left(1-\tfrac{7}{100}\right)p\cdot 9p^2(1-p)^5 + \left(1-\tfrac{9}{100}\right)p\cdot 27p^3(1-p)^6$$

$$+ \left(1-\tfrac{10}{100}\right)p\cdot 81\cdot p^4(1-p)^6 + \left(1-\tfrac{11}{100}\right)p\cdot 162p^5(1-p)^6$$

$$ATI = N(1-P_a) + \sum_{(n,X_n)\in A} n\cdot Pr(\text{ending at } (n,X_n))$$

$$= 100(1-P_a(p)) + 3p^0(1-p)^3 + 5\cdot 3p(1-p)^4 + 7\cdot 9p^2(1-p)^5 + 9\cdot 27\cdot p^3(1-p)^6$$

$$+ 10\cdot 81\cdot p^4(1-p)^6 + 11\cdot 162p^5(1-p)^6$$

5.5

(a). Find single limit variables plan.

$L = 1.000 \quad \sigma = .015 \quad p_1 = .03 \quad P_{a_1} = .95$

$p_2 = .10 \quad P_{a_2} = .10$

$$n \approx \left[\frac{Q_Z(P_{a_1}) - Q_Z(P_{a_2})}{Q_Z(1-p_1) - Q_Z(1-p_2)} \right]^2$$

$$= \left[\frac{Q_Z(.95) - Q_Z(.10)}{Q_Z(1-.03) - Q_Z(1-.10)} \right]^2 = \left[\frac{1.6449 - (-1.2816)}{1.8808 - 1.2816} \right]^2 = 23.85 \approx 24$$

$$\Delta_1 \approx \sigma \left[\frac{Q_Z(P_{a_1})\,Q_Z(1-p_2) - Q_Z(P_{a_2})\,Q_Z(1-p_1)}{Q_Z(P_{a_1}) - Q_Z(P_{a_2})} \right]$$

$$= .015 \left[\frac{(1.6449)(1.2816) - (-1.2816)(1.8808)}{1.6449 - (-1.2816)} \right]$$

$$= .0232$$

Accept sampled lot if for sample of $n=24$, $\bar{X} \geq L + \Delta_1 = 1.000 + .0232 = 1.0232$ and reject the lot ow

$p_1 = .03 = \Phi\left(\frac{1-\mu}{.015}\right) \Rightarrow \mu_1 = 1.0282$

$p_2 = .10 = \Phi\left(\frac{1-\mu}{.015}\right) \Rightarrow \mu_2 = 1.0192$

$$P_a = 1 - \Phi\left(\frac{1 + .0232 - 1.0282}{.015/\sqrt{24}}\right) = .9488$$

$$P_a = 1 - \Phi\left(\frac{1 + .0232 - 1.0192}{.015/\sqrt{24}}\right) = .0957$$

Now pick other μ's around / between μ_1, μ_2 & get p & P_a $\rightarrow$ plot P_a vs. p.

For attributes sampling plan use (8.19) V+J with $\lambda_1 = p_1 = .03$, $\lambda_2 = p_2 = .10$, $P_{a_1} = .95$, $P_{a_2} = .10$

$$\frac{Q_{2(c+1)}(1-P_{a_2})}{Q_{2(c+1)}(1-P_{a_1})} = \frac{\lambda_2}{\lambda_1}$$ then find c, k (see page 457-458 V&J)

and then compare to variables sampling plan #

5.5

(3)

(b). find double limits sampling plan

$L = .49 \quad U = .51 \quad \sigma = .004 \quad p_1 = .03 \quad P_{a1} = .95$

$p_2 = .10 \quad P_{a2} = .10$

$U - L = .51 - .49 = .02 \approx 6\sigma = .024$

(i.e, U-L can be considered "large") or else follow p 471-472 V+J

$\therefore$ use single limit soln. (8.38) & (8.39) V+J.

$$\Rightarrow \begin{cases} n = 23.85 \approx 24 \\ \Delta_2 \approx .006187 \end{cases}$$

Accept sampled lot if for sample of $n = 24$, $L + \Delta_2 \le \bar{x} \le U - \Delta_2$

$\Leftrightarrow .49 + .006187 \le \bar{x} \le .51 - .006187$

or reject the lot ow.

$p_1 = .03 = 1 - \left[\Phi\left(\frac{.51-\mu_1}{.004}\right) - \Phi\left(\frac{.49-\mu_1}{.004}\right)\right] \Rightarrow$ can get μ_1

$p_2 = .10 = 1 - \left[\Phi\left(\frac{.51-\mu_2}{.004}\right) - \Phi\left(\frac{.49-\mu_2}{.004}\right)\right] \Rightarrow$ can get μ_2

by interpolating Table 8.3 V+J

$P_a = \Phi\left(\frac{.51 - .00619 - \mu_1}{.004/\sqrt{24}}\right) - \Phi\left(\frac{.49 + .00619 - \mu_1}{.004/\sqrt{24}}\right) = \#$ $\leftarrow$ can get it using μ_1 above

$P_a = \Phi\left(\frac{.51 - .00619 - \mu_2}{.004/\sqrt{24}}\right) - \Phi\left(\frac{.49 + .00619 - \mu_2}{.004/\sqrt{24}}\right) = \#\#$ $\leftarrow$ " " " μ_2 "

Now pick some other μ's between μ_1 and μ_2 & get p & $P_a \rightarrow$ plot P_a vs p

For attributes sampling plan similarly use (8.19) V+J as before

$$\frac{Q_{2(c+1)}(1-.10)}{Q_{2(c+1)}(1-.95)} = \frac{.10}{.03}$$ and find c & k using (8.20), (8.21),

note that interpolation using Table 8.2 can be used.

[Check back by using formula (8.8)]

Then compare to Variables Sampling plan #

5.5

(c). Use Wallis approximation to find a single limit variable sampling plan for $L = 1.000$, $p_1 = .03$ $\quad P_{a1} = .95$

$p_2 = .10$ $\quad P_{a2} = .10$

$$K \approx \frac{Q_z(P_{a1})\,Q_z(1-p_2) - Q_z(P_{a2})\,Q_z(1-p_1)}{Q_z(P_{a1}) - Q_z(P_{a2})}$$

$$= \frac{(1.6449)(1.2816) - (-1.2816)(1.8808)}{1.6449 - (-1.2816)} = 1.544$$

$$n \approx \left(1 + \frac{K^2}{2}\right)\left[\frac{Q_z(P_{a1}) - Q_z(P_{a2})}{Q_z(1-p_1) - Q_z(1-p_2)}\right]^2$$

$$= \left(1 + \frac{1.544^2}{2}\right)\left(\frac{1.6449 - (-1.2816)}{1.8808 - 1.2816}\right)^2 = 52.2870$$

accept sampled lot if for sample of size 52

$$\bar{x} \geq L + Ks = 1 + 1.544\,s$$

$$p_1 = .03 = \Phi\left(\frac{L-\mu}{\sigma}\right) \Rightarrow \frac{L-\mu}{\sigma} = \Phi^{-1}(.03) = -1.88$$

$$p_2 = .10 = \Phi\left(\frac{L-\mu}{\sigma}\right) \Rightarrow \frac{L-\mu}{\sigma} = \Phi^{-1}(.10) = -1.2816$$

$$P_a = 1 - \Phi\left(\frac{\frac{L-\mu}{\sigma} + K}{\sqrt{\frac{1}{n} + \frac{K^2}{2n}}}\right) = 1 - \Phi\left(\frac{-1.8808 + 1.544}{\sqrt{\frac{1}{52} + \frac{(1.544)^2}{2(52)}}}\right) = 1 - \Phi(-1.6404)$$

$$= .9495$$

$$P_a = 1 - \Phi\left(\frac{-1.2816 + 1.544}{\sqrt{\frac{1}{52} + \frac{(1.544)^2}{2(52)}}}\right) = .1006$$

Now for other values of $\frac{L-\mu}{\sigma}$ around / between -1.88 and -1.2816 find p and P_a. Then plot P_a vs. p to get OC curve.

5.6 find n for known σ single limit variable acceptance sampling plan to have

$$P_a = .95 \text{ if } p = 10^{-6}$$
$$P_a = .10 \text{ if } p = 3\times 10^{-6}$$

$$n \approx \left(\frac{Q_z(P_{a1}) - Q_z(P_{a2})}{Q_z(1-p_1) - Q_z(1-p_2)} \right)^2$$

$$= \left(\frac{Q_z(.95) - Q_z(.10)}{Q_z(1-10^{-6}) - Q_z(1-3\times10^{-6})} \right)^2$$

$$= \left(\frac{1.6449 - (-1.2816)}{4.753 - 4.526} \right)^2 = 166.2055$$

we are discriminating b/w very small fraction nonconforming. We are relying heavily on appropriateness of normal model in the tails of the real dsn. of measurements.

5.9 Variable acceptance sampling based on exp. dist'd observ.
Single Lower limit $L = 0.2107$

(a). $X \sim \exp(\lambda)$, $\lambda > 0$ with mean λ
Var λ^2

$$f_\lambda(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & x > 0 \\ 0 & \text{ow} \end{cases}$$

$$p = P(X < .2107)$$
$$= \int_0^{.2107} \frac{1}{\lambda} e^{-x/\lambda}\, dx = -e^{-x/\lambda} \Big|_0^{.2107} = 1 - e^{-.2107/\lambda}$$

$$\Rightarrow \lambda(p) = \frac{-.2107}{\ln(1-p)}$$

$$\lambda(.10) = 1.9998 \simeq 2$$

$$\lambda(.19) = .9999 \simeq 1$$

5.9 cont'd

(b). Find $n, k \ni$ acceptance sampling plan that rejects a lot if $\bar{x} < k$ has $P_a = .95$ for $p = .10$ and $P_a = .10$ for $p = .19$.

for large n $\quad \bar{x} \dot{\sim} N(\lambda, \frac{\lambda^2}{n})$

$$P(\bar{x} > k) = .95$$

$$P\left(Z > \frac{k-\lambda}{\lambda/\sqrt{n}}\right) = .95$$

$$\Rightarrow \frac{k-\lambda}{\lambda/\sqrt{n}} = -1.645$$

$\lambda(.10) = 2$

$$\Rightarrow \frac{k-2}{2/\sqrt{n}} = -1.645 \quad (*)$$

also $P(\bar{x} > k) = .10$

$$P\left(Z > \frac{k-\lambda}{\lambda/\sqrt{n}}\right) = 1.2816$$

$\lambda(.19) = 1$

$$\Rightarrow \frac{k-1}{1/\sqrt{n}} = 1.2816 \quad (**)$$

from $(*)$ and $(**)$

$$\frac{k-2}{-1.645} = 2\left(\frac{k-1}{1.2816}\right)$$

$$1.2816(k-2) = 2(-1.645)(k-1)$$

$$\Rightarrow k = 1.2803$$

$$\Rightarrow \frac{1.2803-2}{2/\sqrt{n}} = -1.645 \Rightarrow \text{solve for } n$$

or $$\frac{1.2803-1}{1/\sqrt{n}} = 1.2816 \Rightarrow \text{solve for } n$$

$\boxed{5.9}$ cont'd

(c). $P_a = P(\bar{x} > 1.28)$

$$\approx 1 - \Phi\left(\frac{1.28 - \frac{-.2107}{\ln(1-p)}}{\frac{-.2107}{\sqrt{5}\ln(1-p)}} \right)$$

$$= 1 - \Phi\left(\frac{\sqrt{5}\ln(1-p)\left[1.28 + \frac{.2107}{\ln(1-p)}\right]}{-.2107} \right)$$

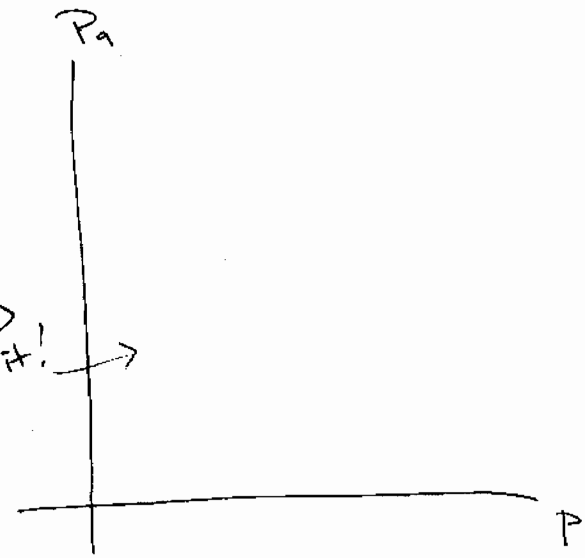
$$= 1 - \Phi\left(\frac{1.28\sqrt{5}\ln(1-p) + .2107\sqrt{5}}{-.2107} \right)$$

$$= 1 - \Phi\left(-13.584 \ln(1-p) - 2.2361 \right)$$

p	P_a
0	.9873
.2	.2133
.4	1.2817×10^{-6}
.1	.7896
.3	.0045
.15	.5115

$\Longrightarrow$

plot it!



5.12

(a) An appropriate expected total cost is

$$ETC = k_1 n + (N-n)(1-P_a)k_1 + (N-n)P_a[p_M k_2 + p_D k_3]$$

$$= Nk_1 + (N-n)P_a[p_M k_2 + p_D k_3 - k_1]$$

It is only the 2nd term that we may influence by choice of n (and the decision criterion). When

$$p_M \cdot k_2 + p_D \cdot k_3 > k_1 \quad (*)$$

this term is positive and we wish to remove it from the ETC. This can be done by choice of $n = N$ ie "All". On the other hand, when

$$p_M k_2 + p_D k_3 < k_1 \quad (**)$$

the term is negative and we wish to make it's multiplier as large as possible. This can be done by choice of $n = 0$ and $P_a = 1$, ie "None".

So in case $(*)$ "all" is optimal, while in case $(**)$ "none" is optimal.

(b) With n inspections made, and counts X_G, X_M and X_D in hand, with rejection one has conditional expected total cost

$$Nk_1 \quad (\star)$$

while with acceptance, the conditional expected total cost is

$$nk_1 + (N-n)\left\{k_2 E[p_M \mid X_G, X_M, X_D] + k_3 \cdot E[p_D \mid X_G, X_M, X_D]\right\} \quad (\star\star)$$

and one should reject the lot if $(\star)$ is less than $(\star\star)$ i.e. if

$$(N-n)k_1 - (N-n)\{\quad\} < 0$$

i.e. if $$k_1 < k_2 \frac{\alpha_M + X_M}{\alpha_G + \alpha_M + \alpha_D + n} + k_3 \frac{\alpha_D + X_D}{\alpha_G + \alpha_M + \alpha_D + n}$$