

P323, 7 The probability of one part failing to meet inspection is equal to the long-run fraction of parts failing to meet inspection, if the part measurements are considered to be identical random variables. You need to make μ small enough so that

$$P(X > 3.150) \leq .03.$$

This is equivalent to $P(X \leq 3.150) > .97$, which is equivalent to $P(Z \leq \frac{3.150 - \mu}{.002}) > .97$, where Z is a standard normal random variable. Looking up .97 in the body of Table B.3

$$\frac{3.150 - \mu}{.002} > 1.88,$$

or $\mu < 3.14624$.

P332, 4.2. (a) The probability of an individual depth being within specifications is the same as the long-run fraction of depths within specifications, if successive shelf depths can be considered identical random variables.

$$\begin{aligned} P(.0275 \leq X \leq .0278) &= P(X \leq .0278) - P(X < .0275) \\ &= P(Z \leq 2) - P(Z < -1) \\ &= .9773 - .1587 = .8186. \end{aligned}$$

(Z is a standard normal random variable.)

(b) Assuming that $\mu = .02765$, we want

$$P(.0275 \leq X \leq .0278) = .98.$$

By symmetry, this is equivalent to $P(X \leq .0278) = .99$, which is equivalent to

$$P\left(Z \leq \frac{.0278 - .02765}{\sigma}\right) = .99.$$

Looking up .99 in the body of Table B.3, this means that

$$\frac{.00015}{\sigma} \approx 2.33$$

or that $\sigma \approx .00006438$.

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P322.3. From P263.1, $EX = \mu = \frac{13}{27}$ and $\text{Var}X = \sigma = .28808$.

(a) Using equation ~~(5.59)~~^{5.53}, $E\bar{X} = \mu = \frac{13}{27}$. Using equation ~~(5.60)~~^{5.56},

$$\begin{aligned}\sqrt{\text{Var}\bar{X}} &= \frac{\sigma}{\sqrt{n}} \\ &= \frac{.28808}{\sqrt{25}} = .05762.\end{aligned}$$

(b) Because n is large and the individual X 's are independent, the central limit theorem says that the distribution of \bar{X} is approximately normal. From part (a), the distribution of \bar{X} is approximately normal with mean $\frac{13}{27}$ and standard deviation .05762.

(c)

$$\begin{aligned}P(\bar{X} > .5) &= 1 - P(\bar{X} \leq .5) \\ &= 1 - P\left(\frac{\bar{X} - \frac{13}{27}}{.05762} \leq \frac{.5 - \frac{13}{27}}{.05762}\right) \\ &= 1 - P(Z \leq .321) \text{ where } Z \text{ is standard normal} \\ &= 1 - .6255 = .3745.\end{aligned}$$

(d)

$$\begin{aligned}P(.4615 \leq \bar{X} \leq .5015) &= P(\bar{X} \leq .5015) - P(\bar{X} < .4615) \\ &= P(Z \leq .35) - P(Z < -.35) \\ &= .6368 - .3632 = .2736.\end{aligned}$$

(e) \bar{X} is approximately normal with mean $\frac{13}{27}$ and standard deviation $\frac{.28808}{\sqrt{100}} = .02881$.

$$\begin{aligned}P(\bar{X} > .5) &= 1 - P(\bar{X} \leq .5) \\ &= 1 - P\left(\frac{\bar{X} - \frac{13}{27}}{.02881} \leq \frac{.5 - \frac{13}{27}}{.02881}\right) \\ &= 1 - P(Z \leq .64) \text{ where } Z \text{ is standard normal} \\ &= 1 - .7389 = .2611.\end{aligned}$$

$$\begin{aligned}P(.4615 \leq \bar{X} \leq .5015) &= P(\bar{X} \leq .5015) - P(\bar{X} < .4615) \\ &= P(Z \leq .69) - P(Z < -.69) \\ &= .7549 - .2451 = .5098.\end{aligned}$$

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P324. 10.

Let X_1, \dots, X_{370} be the 370 individual sheet thicknesses. The thickness of the text is then $U = \sum X_i$. Using equation (5-57), the mean of the sum is the sum of the means:

$$EU = EX_1 + EX_2 + \dots + EX_{370} = 370(.1) = 37.0 \text{ mm.}$$

Assuming that the individual thicknesses are independent, you can use equation (5-58) to say that the variance of the sum is the sum of the variances:

$$\text{Var}U = \text{Var}X_1 + \text{Var}X_2 + \dots + \text{Var}X_{370} = 370(.003)^2 = .00333 \text{ mm}^2,$$

so the standard deviation of U is $\sqrt{.00333} = .0577 \text{ mm.}$

P326. 20.

Since the sample size is large, the central limit theorem says that the distribution of \bar{X} is approximately normal. Using equation (5-59), the mean of \bar{X} is μ and the standard deviation of \bar{X} is $\frac{1}{\sqrt{25}}$.

$$\begin{aligned} P(\mu - .03 \leq \bar{X} \leq \mu + .03) &= P(\bar{X} \leq \mu + .03) - P(\bar{X} < \mu - .03) \\ &= P\left(\frac{\bar{X} - \mu}{\frac{1}{\sqrt{25}}} \leq \frac{.03}{\frac{1}{\sqrt{25}}}\right) - P\left(\frac{\bar{X} - \mu}{\frac{1}{\sqrt{25}}} < \frac{-.03}{\frac{1}{\sqrt{25}}}\right) \\ &= P(Z \leq 1.5) - P(Z < -1.5) \\ &= .9332 - .0668 = .8664. \end{aligned}$$

(Z is a standard normal random variable.)