Stat 305 HW #11 Solutions 1/2 Wu

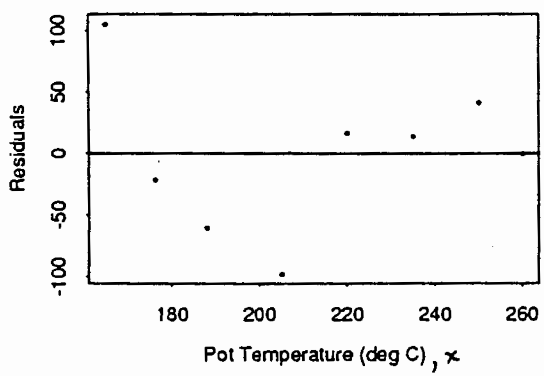
P674. 1 (a) See P140. 3 for the necessary computations. Using equation (9-10),

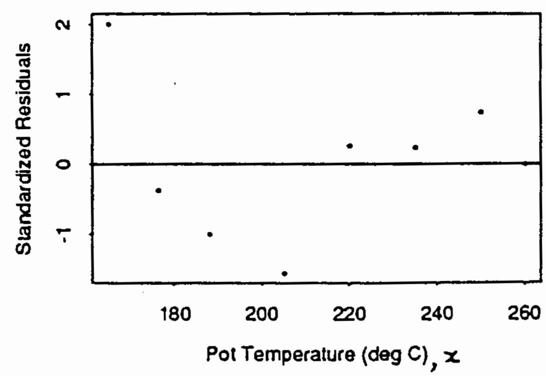
$$s_{\mathrm{LF}}^2 = \frac{1}{8-2}(26940.69) = 4490.116,$$

so $s_{\mathrm{LF}} = \sqrt{4490.116} = 67.01$, with 6 degrees of freedom associated with it. This measures the baseline variation in molecular weight that would be observed for any fixed pot temperature, assuming that model (9-4) is appropriate.

(b) The residuals were computed in P140. 3 . Use equation (9-12) to compute the standardized residuals. $\bar{x} = 212.375$, and $\sum(x - \bar{x})^2 = 8469.875$. The rest of the calculations are summarized below.

x	$\sqrt{1 - \frac{1}{8} - \frac{(x-212.375)^2}{8469.875}}$	e	e^*
165	.78103	105.35535	2.01306
176	.84781	-21.12558	-.37186
188	.89714	-60.10477	-.99982
205	.93198	-97.57529	-1.56245
220	.93174	16.95072	.27150
235	.90253	14.47673	.23938
250	.84135	42.00275	.74503
260	.77924	.02009	.00038





The plots look almost exactly the same.

(c) This is β_1. Use equation (9-17). For 90% confidence, the appropriate t is $t = Q_6(.95) = 1.943$ from Table B-4. The resulting interval is

$$\begin{aligned} & 23.49827 \pm 1.943\frac{67.01}{\sqrt{8469.875}} \\ = \; & 23.49827 \pm 1.414696 \\ = \; & [22.08, 24.91]. \end{aligned}$$

(d) Use equation (9-24). The appropriate t is the same as the one in part (c). The resulting interval for the mean at $x = 212$ is

$$\begin{aligned} & 1807.063 \pm 1.943(67.01)\sqrt{\frac{1}{8} + \frac{.140625}{8469.875}} \\ = & \ 1807.063 \pm 46.03471 \\ = & \ [1761.03, 1853.10]. \end{aligned}$$

The resulting interval for the mean at $x = 250$ is

$$\begin{aligned} & 2699.997 \pm 1.943(67.01)\sqrt{\frac{1}{8} + \frac{1415.641}{8469.875}} \\ = & \ 2699.997 \pm 70.37134 \\ = & \ [2629.63, 2770.37]. \end{aligned}$$

(e) Use equation (9-25). The appropriate f is $f = Q_{2,6}(.90) = 3.46$ from Table B-6-B. The resulting interval for the mean at $x = 212$ is

$$\begin{aligned} & 1807.063 \pm \sqrt{2(3.46)}(67.01)\sqrt{\frac{1}{8} + \frac{.140625}{8469.875}} \\ = & \ 1807.063 \pm 62.32548 \\ = & \ [1744.74, 1869.39]. \end{aligned}$$

The resulting interval for the mean at $x = 250$ is

$$\begin{aligned} & 2699.997 \pm \sqrt{2(3.46)}(67.01)\sqrt{\frac{1}{8} + \frac{1415.641}{8469.875}} \\ = & \ 2699.997 \pm \\ = & \ [2604.72, 2795.27]. \end{aligned}$$

(f) Use equation (9-26). For a 90% one-sided interval, appropriate t is $t = Q_6(.90) = 1.440$ from Table B-4. The resulting lower prediction bound at $x = 212$ is

$$\begin{aligned} & 1807.063 - 1.440(67.01)\sqrt{1 + \frac{1}{8} + \frac{.140625}{8469.875}} \\ = & \ 1807.063 - 102.346 \\ = & \ 1704.72. \end{aligned}$$

The resulting bound for the mean at $x = 250$ is

$$\begin{aligned} & 2699.997 - 1.440(67.01)\sqrt{1 + \frac{1}{8} + \frac{1415.641}{8469.875}} \\ = & \ 2699.997 - 109.6846 \\ = & \ 2590.31. \end{aligned}$$

(h) Using the general form given in Table 9-6.

Source	SS	df	MS	F
Regression	4676798	1	4676798	1041.58
Error	26941	6	4490	
Total	4703739	7		

The p-value is

$$P(\text{an } F_{1,6} \text{ random variable} > 1041.58),$$

which is less than .001, according to Tables B-6 ($Q_{1,6}(.999) = 35.51$). This is overwhelming evidence that the mean average molecular weight is related to the pot temperature. (The model used is an improvement over the model $y = \beta_0 + \epsilon$.)