

Name _____

Note: Points for each question are indicated in the left margin in square brackets.

1. The data used in creating the attached one page of JMP output come from a study on the hardness of a certain type of plastic washers used in the manufacture of roofing nails. The variables studied were

x_1 (time): the time the plastic is allowed to cure,

x_2 (component (%)): the percentage of a chemical component in the plastic,

and the response variable was

y (hardness): the hardness of the washer.

- (a) What is the sample correlation between y and x_1 ? (Give a number.)

[3]
$$\sqrt{R^2} = \sqrt{0.7756} \approx 0.881.$$

Use the fitted equation in x_1 and x_2 to answer the following questions (b) to (f).

- (b) What is the fraction of observed raw variation in y accounted for by a linear equation in x_1 and x_2 ? (Give a number.)

[3]
$$R^2 = 0.974$$

- (c) What is the sample correlation between y and \hat{y} ? (Give a number.)

[3]
$$\sqrt{R^2} = \sqrt{0.974} \approx 0.987$$

- (d) Find the residuals for the hardness of the washer for the second and third data points.

[6] 2nd data point: $y - \hat{y} = 297 - [229.81 + 1.1741 \times 34 + 0.536 \times 53]$
 $= 297 - 298.14 = -1.14$

3rd data point: $y - \hat{y} = 264 - [229.81 + 1.1741 \times 12 + 0.536 \times 44] = -3.48$

- (e) Within the range of $12 \leq x_1 \leq 60$ and $15 \leq x_2 \leq 53$, find the minimum and maximum predicted hardness of the washer.

[8] Minimum predicted $\hat{y} = 229.81 + 1.1741 \times 12 + 0.536 \times 15$
 $= 257.94$

Maximum $\hat{y} = 229.81 + 1.1741 \times 60 + 0.536 \times 53 = 328.66$

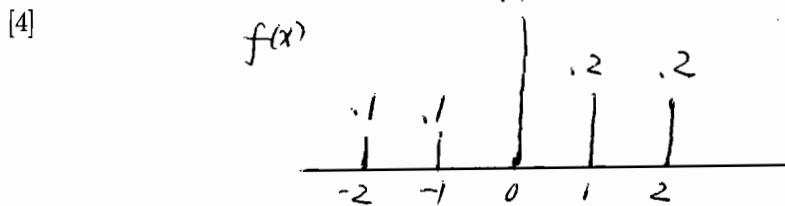
- (f) About what change in average hardness would be if both x_1 (time) and x_2 (component (%)) are increased by 2 units?

[7]
$$\Delta \hat{y} = 1.1741 \times 2 + 0.536 \times 2 = 3.42$$

2. Consider a discrete random variable X with the probability function as specified below.

| | | | | | |
|--------|----|----|----|----|----|
| x | -2 | -1 | 0 | 1 | 2 |
| $f(x)$ | .1 | .1 | .4 | .2 | .2 |

- (a) Make a spike graph for X .



- (b) Find the cumulative probability function for X .

[5]

$$F(x) = P(X \leq x) = \begin{cases} 0 & x < -2 \\ .1 & -2 \leq x < -1 \\ .2 & -1 \leq x < 0 \\ .6 & 0 \leq x < 1 \\ .8 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

- (c) Find the mean and standard deviation of X .

[5]

$$E X = \sum x f(x) = (-2)(.1) + (-1)(.1) + 0(.4) + 1(.2) + 2(.2) = 0.3$$

$$E(X - EX)^2 = \sum (x - EX)^2 f(x) = (-2 - .3)^2 (.1) + (-1 - .3)^2 (.1) + (0 - .3)^2 (.4) + (1 - .3)^2 (.2) + (2 - .3)^2 (.2) = 1.41$$

$$\sqrt{E(X - EX)^2} = \sqrt{1.41} \approx 1.187$$

3. Suppose that 10% of light bulbs produced by a certain machine are defective. 10 such light bulbs are randomly selected. What is the probability that at least 8 of them are good?

[6] X : number of good light bulbs. X : Binomial (10, 0.9)

$$P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$$

$$= \frac{10!}{8! \cdot 2!} (0.9)^8 (0.1)^2 + \frac{10!}{9! \cdot 1!} (0.9)^9 (0.1)^1 + \frac{10!}{10! \cdot (10-10)!} (0.9)^{10}$$

$$= 0.19371 + 0.38742 + 0.34868 \approx 0.9298$$

| y | x1 | x2 |
|-----|----|----|
| 276 | 22 | 36 |
| 297 | 34 | 53 |
| 264 | 12 | 44 |
| 290 | 40 | 22 |
| 276 | 34 | 17 |
| 293 | 44 | 25 |
| 323 | 60 | 45 |
| 293 | 40 | 25 |
| 276 | 32 | 15 |
| 291 | 27 | 46 |

Response y

Whole Model

x1

Summary of Fit

| | |
|----------------------------|----------|
| RSquare | 0.775563 |
| RSquare Adj | 0.747509 |
| Root Mean Square Error | 8.16596 |
| Mean of Response | 287.9 |
| Observations (or Sum Wgts) | 10 |

Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob> t |
|-----------|-----------|-----------|---------|---------|
| Intercept | 249.98729 | 7.659152 | 32.64 | <.0001* |
| x1 | 1.0989191 | 0.209006 | 5.26 | 0.0008* |

Response y

Whole Model

x1

x2

Summary of Fit

| | |
|----------------------------|----------|
| RSquare | 0.974016 |
| RSquare Adj | 0.966592 |
| Root Mean Square Error | 2.970371 |
| Mean of Response | 287.9 |
| Observations (or Sum Wgts) | 10 |

Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob> t |
|-----------|-----------|-----------|---------|---------|
| Intercept | 229.81246 | 3.921124 | 58.61 | <.0001* |
| x1 | 1.174067 | 0.076718 | 15.30 | <.0001* |
| x2 | 0.5360435 | 0.073312 | 7.31 | 0.0002* |

Stat 305

Quiz 2

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y: hardness

x1: time

x2: component (%)