Extraction of a Multiscale Structure of a Turbulent Lobed Jet from PIV Results Using a Vector Wavelet Multiresolution Technique

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The wavelet-based vector multiresolution technique was applied to the postprocessing of PIV results for identifying the multiscale turbulent structures of a lobed jet. By analyzing the instantaneous streamlines and vorticity fields of various scales, we obtained important information on the multiscale flow structures. The large-scale vortices or coherent structures with a central scale of $a = 15.36 \text{ mm}$ were easily extracted from PIV measurement results. The intermediate- and small-scale vortices embedded within the large-scale vortices were separated and visualized.

Key Words: Orthogonal Vector Wavelet Transform, Particle Image Velocimetry, Wavelet Multiresolution Analysis, Multiscale Vortex, Turbulent Lobed Jet

1. Introduction

The turbulent jet exhibited a complex structure with a wide range of coexisting scales and a variety of shapes in the dynamics, and its coherent structures were responsible for most of the momentum, mass and heat transfer. During the past couple of years, the development of PIV techniques has made it possible to provide more detailed information on flow structures, such as the instantaneous values of various flow quantities and their distribution and transient variation. Despite the usefulness of information we obtained by examining the measured instantaneous fields and the time-mean turbulent quantities, further information on the instantaneous multiscale structures has not yet been clarified.

In the past decade, wavelet transform has been widely applied to the analysis of turbulent flows.1–10) This technique allows the tracking of turbulent structures in terms of time and scale and extracts new information on turbulent structures of various scales. To gain deeper insight into the multiscale structures and coherent structures in the turbulent flows, it is important to analyze the instantaneous full flow field. Recently, Li et al.9,10) applied the two-dimensional orthogonal wavelets to the turbulent images for the identification of the instantaneous turbulent structures of various scales. However, the research concerning the extraction of instantaneous turbulent structures of various scales from PIV data is still open.

The purpose of this study is to employ the vector wavelet multiresolution technique to the postprocessing of PIV results for revealing and visualizing the instantaneous multiscale turbulent structures in a lobed jet.

2. Vector Wavelet Multiresolution Technique

For a two-dimensional vector field $\vec{f}(x_1, x_2)$ and a wavelet basis $\psi_m; n_1, n_2(x_1, x_2)$, the two-dimensional discrete wavelet transform is defined by

$$\tilde{Wf}_{m; n_1, n_2} = \sum_{i} \sum_{j} f(x_1^i, x_2^j)\psi_{m; n_1, n_2}(x_1^i, x_2^j).$$ (1)

The reconstruction of the original vector field can be achieved by using

$$\tilde{f}(x_1, x_2) = \sum_{m} \sum_{n_1} \sum_{n_2} \tilde{Wf}_{m; n_1, n_2}\psi_{m; n_1, n_2}(x_1, x_2).$$ (2)

The two-dimensional wavelet basis, $\psi_{m; n_1, n_2}(x_1, x_2)$, is simply to take the tensor product functions generated by two one-dimensional bases as

$$\psi_{m; n_1, n_2}(x_1, x_2) = \psi_m(2^{-m}x_1 - n_1)\psi_m(2^{-m}x_2 - n_2).$$ (3)

The oldest example of a function $\psi(x)$ for which the $\psi_{m, n}(x)$ constitutes an orthogonal basis is the Haar function, constructed long before the term “wavelet” was coined. In the past ten years, various orthogonal wavelet bases have been constructed, for example, Meyer, Daubechies, Coifman, Battle-Lemarie, Baylkin, and spline. They provide excellent localization properties in physical and frequency spaces. In this study we use the Daubechies basis with order 20, which is not only orthonormal, but it also has smoothness and compact support.

Since the discrete wavelet transform is a linear transform, the wavelet multiresolution analysis is used to completely decompose a nonlinear problem into the superposition of a limited number of grouped scale components (from small-scale to large-scale). Thus Eq. (8) can be written in the fol-
lowing equivalent form:
\[ \vec{f}(x_1, x_2) = \vec{f}w_1 + \vec{f}w_2 + \cdots + \vec{f}w_k \]  
(4)

where
\[ \vec{f}w_i = \sum_{n_1} \sum_{n_2} Wf_{i,n_1,n_2} \psi_{i,n_1,n_2}(x_1, x_2) \]  
(5)

On the right side of Eq. (4), the first term \( \vec{f}w_1 \) and the last term \( \vec{f}w_k \) represent the components at level 1 (the largest scale) and level \( k \) (the smallest frequency).

The procedure of the vector wavelet multiresolution analysis can be summarized in the following two steps:

1. The discrete wavelet coefficients \( Wf_{m,n_1,n_2} \), representing vector field in orthonormal wavelet basis, are computed by using the discrete wavelet transform of Eq. (1).
2. Inverse discrete wavelet transform of Eq. (5) is applied to wavelet coefficients at each wavelet level, and the components of vector field are obtained at each wavelet level or scale.

It is evident that a sum of all vector components can recover the original vector field. Therefore the vector wavelet multiresolution analysis may process fewer data by selecting the relevant details necessary to perform an extraction of the multiscale structures, it may decompose the vector data in both Fourier and physical spaces.

3. Experimental Procedures

The experiment was carried out in liquid-phase turbulent-jet flows. A test lobed nozzle with six lobes, as shown in Fig. 1, is used in the present study. The width of each lobe is 6 mm and the height is 15 mm (\( H = 15 \text{ mm} \)). The inner and outer penetration angles of the lobed structures are \( \theta_{\text{in}} = 22^\circ \) and \( \theta_{\text{out}} = 14^\circ \), respectively. The equivalent nozzle diameter, which is defined as the diameter of the circle with the same area as the lobed nozzle exit area, is designed to be \( d = 40 \text{ mm} \). The z-axis is taken as the direction of the main stream; the \( x-y \) plane is perpendicular to the z-axis and is taken as the cross plane of the lobed jet. The core jet velocity (\( U_0 \)) was set at about 0.1 m/s in the present study. The Reynolds number of the jet flow is about 3,000 based on the equivalent nozzle diameter and the core jet velocity.

The PIV measurement was supplied by a double-pulsed Nd:YAG Laser sheet (with a thickness of about 1.0 mm) at a frequency of 10 Hz and a power of 200 mJ/pulse. The time interval between the two pulses can be adjustable, which is about 2 to 5 ms for the present study. The PIV images of transverse sections at downstream position \( z = 40 \text{ mm} \) were captured by a 1,008 x 1,016 pixels Cross-Correlation CCD array camera. The spatial resolution of the PIV images for the present research case is about 120 \( \mu \text{m/pixel} \). The images were divided into 8 by 8 pixel interrogation windows, and 50% overlap grids were employed. The Hierarchical Recursive PIV method was used in the present study for obtaining velocity vectors of about 225 x 225. The postprocessing procedures, which include the subpixel interpolation and spurious velocity deletion, were adopted to improve the accuracy of the PIV results. More details have been given in Hu et al.\cite{11}

4. Results and Discussion

Figure 2 shows original instantaneous streamlines of a cross plane at downstream position \( z/d = 1.0 \) in a lobed mixing jet with \( Re = 3,000 \). The irregular flow structures that imply multiscale structures can be observed. The irregular large-scale vortices, i.e., coherent structure, can be seen around the positions of lobe peak.

For identifying the multiscale flow structures, the velocity vector field obtained from the PIV measurement is decomposed into several wavelet components with different wavelet levels or broader scales based on the vector wavelet multiresolution technique. Since the measured velocity vectors of 225 x 225 are analyzed in this study, the five wavelet components are obtained presently. The measured velocity vector \( \vec{u}(x, y) \) can be written as the sum of five velocity vector compositions \( \vec{u}_j(x, y) \), i.e.,

\[ \vec{u}(x, y) = \sum_{j=1}^{5} \vec{u}_j(x, y). \]  
(6)

where \( j \) denotes the wavelet level or scale.

![Fig. 1. The test lobed nozzle.](image1)

![Fig. 2. Original instantaneous streamlines of PIV results in a lobed mixing jet at \( Re = 3,000 \).](image2)
Wavelet level 1, which corresponds to the central scale of 15.36 mm, represents the large-scale structure based on an analysis of the power spectrum (not shown) of velocity signal.

The streamlines of various scales are computed from the corresponding wavelet components of the velocity vector. Three wavelet components of instantaneous streamlines with different broader scales are shown in Fig. 3. Figure 3(a) exhibits clearly the large-scale structures with a central scale of \( a = 15.36 \) mm. The large-scale vortices can be clearly seen around the lobe peak regions. These vortices correspond quite well to the vortices in Fig. 2. This agreement provides a validation for the vector wavelet multiresolution technique. The flow structure with a central scale of \( a = 7.68 \) mm is shown in Fig. 3(b). Several intermediate-scale vortices appear at the position of lobe peak and exist in the large-scale vortices of \( a = 15.36 \) mm (in Fig. 3a). Note that such structure cannot be extracted by the traditional techniques. Figure 3(c) shows the flow structure with a central scale of \( a = 3.84 \) mm. Several smaller-scale vortices are identifiable. These vortices seem to occur throughout the position of the lobe.

For even smaller-scale flow structures, it is difficult to describe such structures by using the instantaneous streamlines. We adopt the vorticity to discuss the vortical structures. Figure 4 illustrates a vorticity contour obtained from the original instantaneous PIV velocities that corresponds to the instantaneous streamlines of Fig. 2. The positive and negative vorticities are denoted by solid and dashed lines, respectively. Only a few small peaks in the vorticity field that represents the smaller-scale vortices can be observed in Fig. 4.

To extract the distribution of the multiscale vorticity field, the wavelet components of the velocity vector obtained by using the vector wavelet multiresolution technique are employed to compute the vorticity field. The wavelet components of streamwise vorticity contour with five broader scales are displayed in Fig. 5 and provide information on the distribution of multiscale vortices in a lobed mixing turbulent jet. Several alternative larger positive and negative peaks can be clearly seen at the positions of the lobe peak in Fig. 5(a) and indicate pairs of large-scale streamwise vortices with a central scale of \( a = 15.36 \) mm. They are the uppermost and coherent structures. These larger positive and
negative peaks are consistent with the large-scale vortices observed in the instantaneous streamlines of Fig. 3(a).

In the vorticity component of scale $a = 7.68$ mm (Fig. 5b), several pairs of positive and negative peaks appear at the position of each lobe. They imply pairs of intermediate-scale streamwise vortices. By comparing Fig. 5(a), it is found that each pair of intermediate-scale streamwise vortices is contained in a larger-scale vortex. With a decrease in the scale to $a = 3.84$ mm, as shown in Fig. 5(c), several stronger positive or negative peaks are mainly distributed in the neighborhood of the lobe and correspond to the vortices at this scale range. When the scale

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Fig. 5. Instantaneous multiscale vorticity of PIV results in a lobed mixing jet at $Re = 3,000$. 
is further decreased to $a = 1.92$ and $0.96$ mm, as shown in Fig. 5(d) and (e), a clear distribution of several smaller positive or negative peaks can be observed at the position of the trailing edge geometry of the lobed nozzle. They are much more active than those in the other region. The turbulence appears to become more intense, suggesting that the vortex merging process is an unsteady interaction and that the breakdown of the large-scale streamwise vortices is accompanied by a significant increase in turbulent intense and small-scale vortices. Note that it is impossible to identify such multiscale flow structures by analyzing the original vorticity field of PIV results.

Although preliminary, the results present here demonstrated that the vector wavelet multiresolution technique could be used effectively for the decomposition and analysis of multiscale turbulent structures.

5. Conclusion

The wavelet-based vector multiresolution technique was applied to PIV data in this paper. The components of instantaneous streamlines and vorticity with different scales were obtained, and important information on the multiscale flow structures of a lobed jet was provided. The large-scale vortices or coherent structures with a central scale of $a = 15.36$ mm were extracted from PIV measurement results. The intermediate- and small-scale vortices embedded within the large-scale vortices were also visualized.

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References