

# Visualization of Multi-scale Turbulent Structure in Lobed Mixing Jet Using Wavelets

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**Abstract:** The two-dimensional orthogonal wavelet transform was applied to the LIF image of lobed mixing jet for identifying the multi-scale turbulent structures. The digital imaging slice photographs at  $z/D = 1.0$  and  $1.5$  with  $Re = 3000$  were respectively decomposed into seven image components with different broad scales. These image components provided the visualized information on the multi-scale structures in a lobed mixing turbulent jet. The cores and edges of the vortices and the coherent structures at different resolutions or scales can be easily extracted. It was found that the scale range of the coherent structure becomes narrow along the downstream direction. The size of the intermediate- and small-scale structures does not vary significantly with downstream distance.

**Keywords:** digital image processing, lobed mixing jet, multi-scale vortex, two-dimensional orthogonal wavelet transform, wavelet multi-resolution analysis.

## 1. Introduction

The lobed mixing jets are often used as an important device in the fluid machinery for their efficient mixing enhancement performance and noise reduction (Crouch et al., 1977; Paterson, 1984). It is now a well-known fact that the lobed mixing jet exhibited complex structure with a wide range of coexisting scales and a variety of shapes in the dynamics. Its coherent structures were responsible for most of the momentum, mass and heat transfer. Many identification techniques, such as image processing, spectra analysis, spatial correlation functions, proper orthogonal decomposition, stochastic estimation, and pattern recognition, were well established to understand the vortical dynamic mechanism and turbulent structures. However, these traditional analytical techniques cannot provide us with sufficient or detailed information. The complex vortical and turbulent structures in terms of space, scale and strength have not yet been clarified.

To extract a time-frequency localization of multi-scale turbulent structures, wavelet analysis was applied to the complex turbulent structures since 1989. The great progress of the wavelet applications in the fluid mechanics has been made. Li et al. (1995 and 2000) employed one-dimensional continuous and discrete wavelet transforms to analyze the experimental velocity signals of turbulence in the dimensions of time and frequency. Li (1997a and b; 1998a and b) proposed the wavelet correlation method and wavelet spatial statistics based on wavelet transform, and revealed the multi-scale structure of eddy motion in the turbulent shear flow in both Fourier and physical spaces. These researches indicated that the wavelet technique offered the potentials extracting new information from the turbulent field; however, they limited to the analysis of turbulent structure based on the one-dimensional

wavelet transform.

To gain deeper insight into multi-scale structures and coherent structures in turbulent flows, it is important to analyze the full field by the image processing. Although there were several researches (Everson and Sirovich, 1990; Brasseur and Wang, 1992; Spedding et al., 1993; Dallard and Browand, 1993; Dallard and Spedding, 1993; Kailas and Narasimha, 1999) that applied two or three-dimensional wavelet transform to full field measurements or simulation data, they have thus far concerned the continuous wavelet transform. In spite of that the coefficients of continuous wavelet transform may extract the characterization of local regularity, it is unable to reconstruct the original function because the mother wavelet function is a non-orthogonal function. In the turbulent image processing it is of great significance to study the image components of various scales that can reconstruct the original image based on the orthogonal wavelet transform. Recently, Li et al. (1999) developed an application of two-dimensional orthogonal wavelets to the turbulent images for the identification of the multi-scale turbulent structures.

The aim of this paper is to apply the two-dimensional orthogonal wavelets, i.e. wavelet multi-resolution analysis, to the digital imaging photographs of lobed mixing jets in order to reveal the multi-scale turbulent structures and to extract the most essential scales governing lobed mixing enhancement process.

## 2. Two-dimensional Orthogonal Wavelets

For a two-dimensional scalar field  $f(x_1, x_2)$ , the two-dimensional discrete wavelet transform is defined by

$$Wf_{m_1, n_1; m_2, n_2} = \sum_i \sum_j f(x_1^i, x_2^j) Y_{m_1, n_1; m_2, n_2}(x_1^i, x_2^j) \quad (1)$$

The reconstruction of the original scalar field can be achieved by using

$$f(x_1, x_2) = \sum_{m_1} \sum_{m_2} \sum_{n_1} \sum_{n_2} Wf_{m_1, n_1; m_2, n_2} Y_{m_1, n_1; m_2, n_2}(x_1, x_2) \quad (2)$$

where the two-dimensional wavelet basis,  $Y_{m_1, n_1; m_2, n_2}(x_1, x_2)$ , is simply to take the tensor product functions generated by two one-dimensional bases as

$$Y_{m_1, n_1; m_2, n_2}(x_1, x_2) = 2^{-(m_1 + m_2)/2} \mathcal{Y}(2^{-m_1} x_1 - n_1) \mathcal{Y}(2^{-m_2} x_2 - n_2). \quad (3)$$

The oldest example of a function  $\mathcal{Y}(x)$  for which the  $\mathcal{Y}_{m,n}(x)$  constitutes an orthogonal basis is Haar function, constructed long before the term "wavelet" was coined. In the last ten years, various orthogonal wavelet bases, such as Meyer basis, Daubechies basis, Coifman basis, Battle-Lemarie basis, Baylkin basis, spline basis etc., have been constructed. They provide excellent localization properties both in physical space and frequency space. In this study we use the Daubechies basis with index  $N = 20$ , which is not only orthonormal, but also have smoothness and compact support, to analyze the flow image.

The procedure of the wavelet analysis, i.e. wavelet multi-resolution analysis, can be summarized in two steps:

- (1) Wavelet coefficients or wavelet spectrum of an image is computed based on the discrete wavelet transform of Eq. (1).
- (2) Inverse wavelet transform of Eq. (2) is applied to wavelet coefficients at each wavelet level, and image components are obtained at each level or scale in the wavelet spaces.

The detail regarding the wavelet multi-resolution analysis can be found in Li et al. (1999).

## 3. Experimental Procedures

The present experiment was carried out in liquid-phase turbulent-jet flows and the experimental set-up is shown in Fig. 1. The test nozzles were installed in the middle of the water tank (600 mm × 600 mm × 1000 mm). Fluorescent dye (Rhodamine B) for LIF tracers (polystyrene particles of  $d = 20\text{-}30 \mu\text{m}$ , density is  $1.02 \text{ kg/cm}^3$ ) was premixed with water in jet supply tank and jet flow was supplied by a pump. A convergent unit and honeycomb structures were installed at the inlet of test nozzle to insure the turbulence intensity levels of jet flows at the exit of test nozzle were less than 3%. A flow meter was employed to measure the flow rate of the jet, which was used to

calculate the representative velocity and Reynolds numbers. The pulsed-laser sheet (of thickness about 1.0 mm) used for LIF visualization was supplied by twin Nd:YAG Lasers with the frequency of 10 Hz and power of 200 mJ per pulse. The LIF images of slices were captured by a  $1008 \times 1016$  pixels Cross-correlation CCD array camera (PIVCAM 10-30), and were digitized by an image processing board. The field of view spans  $l_0 \cong 12$  cm, resulting in a pixel resolution of  $l_p \cong 120$   $\mu$ m.

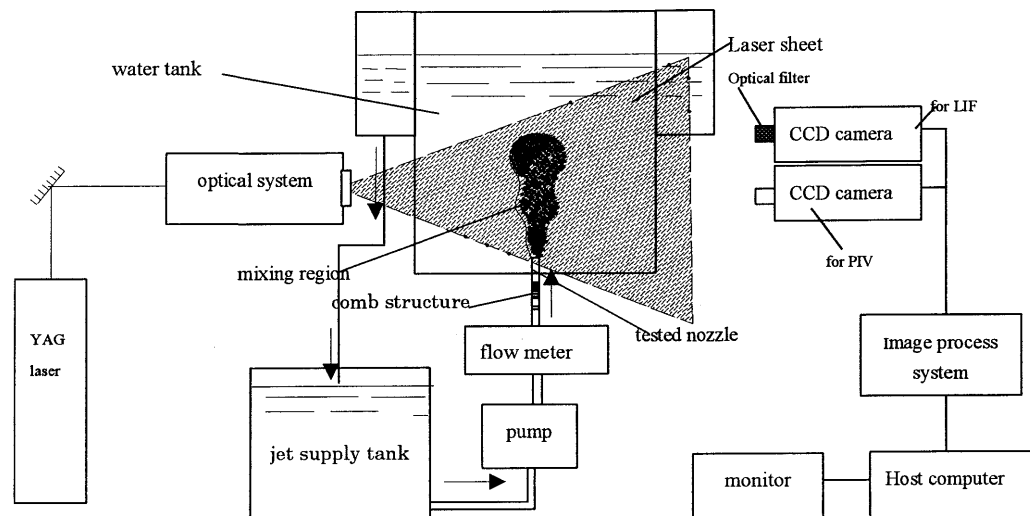


Fig. 1. The schematic of the experimental set-up.

A lobed nozzle with six lobes, as shown in Fig. 2, was used as a test nozzle in the present study. The height of the lobes is 15 mm ( $H = 15$  mm) and the equivalent diameter is  $D = 40$  mm. The inner and outer penetration angles of the lobe structures are  $22^\circ$  and  $14^\circ$  respectively. In the present study, the core jet velocity ( $U_0$ ) was set as 0.1 m/s. The Reynolds number of the jet flow is about 3000 based on the equivalent nozzle diameter ( $D$ ) and the core jet velocity ( $U_0$ ).

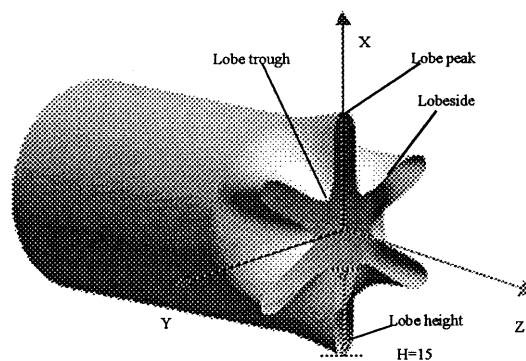


Fig. 2. The test lobed nozzle.

It had been found that the intensive mixing of the core jet flow with ambient flows could be achieved in the very near field of a jet flow by using a lobed nozzle/mixer. Therefore, the CCD camera view of the present study was focused on the near region ( $z/D < 3.0$ ) of the jet mixing flows and the transverse sections of flow field at two downstream positions of  $z = 40$  and 60 mm were visualized and discussed.

## 4. Results and Discussion

An original diametrical slice image of a lobed mixing jet with  $Re = 3000$  at a downstream location of  $z/D = 1.0$  is shown in Fig. 3. It is evident that the large-scale vortices in the shear layer dominate the whole flow structure. The regions of the higher strain (edges of the vortices) are identified as the higher gradient of concentration. To extract the multi-scale turbulent structures, the wavelet multi-resolution analysis is employed to decompose this

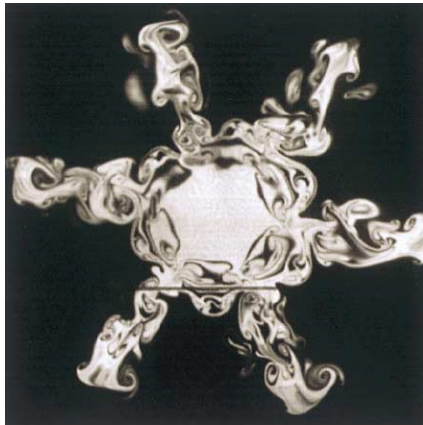


Fig. 3. Original LIF image of a lobed mixing jet at  $z/D = 1.0$ .



Fig. 4. Wavelet coefficients of LIF image.

image. The wavelet coefficients of the original image are first computed based on the two-dimensional orthogonal wavelet transform. The distribution of wavelet coefficients is plotted in Fig. 4. It is evident that the larger magnitudes of wavelet coefficients concentrate on a small area of the bottom left-hand corner, i.e. the larger-scale range. This implies that the large-scale motions dominate the turbulent structures. Then, the image components at seven different broad scales are reconstructed from the wavelet coefficients based on the inverse wavelet transform, and are displayed in Figs. 5(a)-(g). In Figs. 5(a)-(g), the false-colors have been assigned to the scalar values of the image components. The highest concentration is displayed as deep red and the lowest as purple. These images can provide information on the multi-scale structures in a lobed mixing turbulent jet and several features of the turbulent structure may be immediately noted in Fig. 5. At the smallest scale of  $a = 0.24$  mm shown in Fig. 5(a), the whole field appears to be covered by a nearly homogeneous distribution of very fine scale turbulent structures. At the central scales of  $a = 0.48$  and  $0.96$  mm, as shown in Figs. 5(b) and (c), a clear distribution of the edges of the vortices with a smaller-scale can be observed in the interior of the flow except the center region. The horizontal noisy lines appeared in the image components are caused by the background noise of the original image. As the central scale increases to  $a = 1.92$  mm in Fig. 5(d), the edges of the vortices within this scale range can be clearly observed, which are the "zoom-out" image of  $a = 0.96$  mm. At the larger central scales of  $a = 3.84$  and  $7.68$  mm shown in Figs. 5(e) and (f), the core of the vortex is seen as a region of the higher negative concentration (displayed as purple) distributed in the shear layer. The higher concentration appearing in these two image components indicates that the vortices within these two broad scales are more active and dominate the turbulent mixing processes, which are referred to as the coherent structure of the problem. The horizontal noisy lines existing in Figs. 5(a)-(d) disappear within the larger-scale range. This indicates that wavelet multi-resolution analysis can also be used to reduce the noise in an image. At the largest scale of  $a = 15.36$  mm (Fig. 5(g)), the large-scale flow field may be clearly identified by the green region. By comparing with the original image, the positive and negative peaks indicate the position of large-scale vortices except large positive peaks on the center.

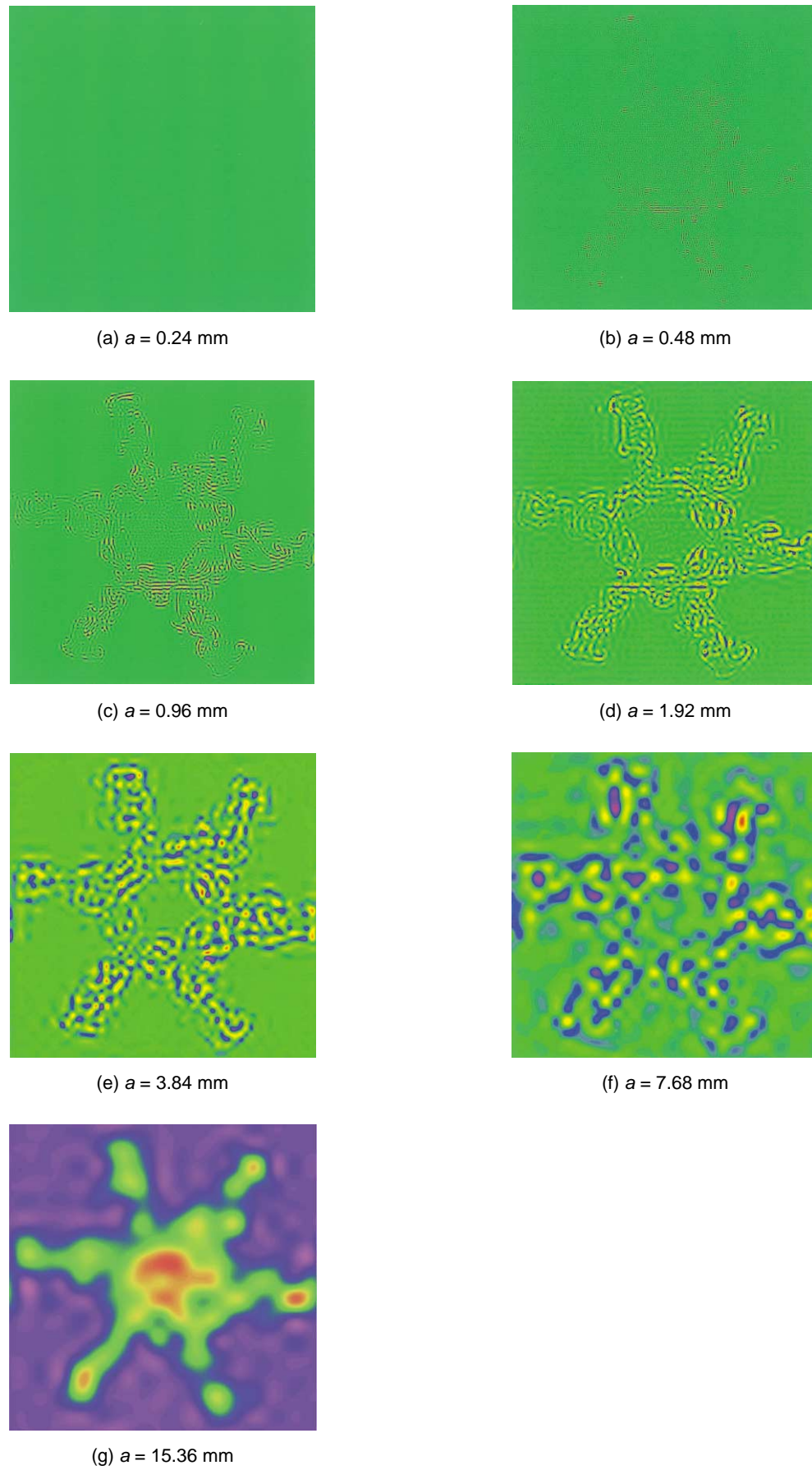


Fig. 5. Multi-resolution images of a lobed mixing jet at  $z/D = 1.0$ .

At further downstream location of  $z/D = 1.5$ , an original cross slice image of a lobed mixing jet with  $Re = 3000$  is shown in Fig. 6. The multi-scale vortices can be clearly seen in the original image. The multi-resolution images at seven different broad scales are shown in Figs. 7(a)-(g). Within the range of small and intermediate scale of  $a = 0.24 \sim 1.92$  mm, the distributions of concentration strength at  $z/D = 1.5$  in Figs. 7(a)-(d), are similar to those at  $z/D = 1.0$  in Figs. 5(a)-(d). This indicates that the size of the intermediate- and small-scale structures does not vary significantly with the downstream distance. A clear distribution of the edges of the vortices in Fig. 7(d) can also be observed in the interior of the flow except the center region. However, at the scale of  $a = 3.84$  mm the concentration of the vortex cores, as shown in Fig. 7(e), becomes lower than that in Fig. 5(e). The higher concentration of the vortex cores appearing in Fig. 7(f) indicates that the stronger vortices within this broad scale, i.e. the coherent structure, are more active and dominate the turbulent mixing process. This implies that the scale range of the coherent structure becomes narrow as increasing the downstream distance. At the largest scale of  $a = 15.36$  mm (Fig. 7(g)), the large-scale flow field which is similar to that in Fig. 5(g) may also be clearly identified by the green region.

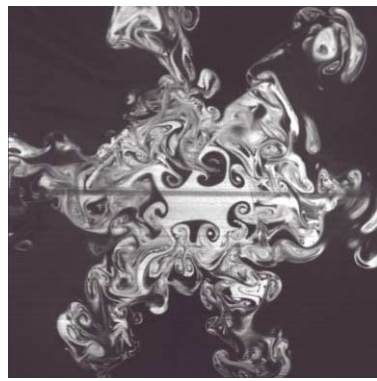


Fig. 6. Original LIF image of a lobed mixing jet at  $z/D = 1.5$ .

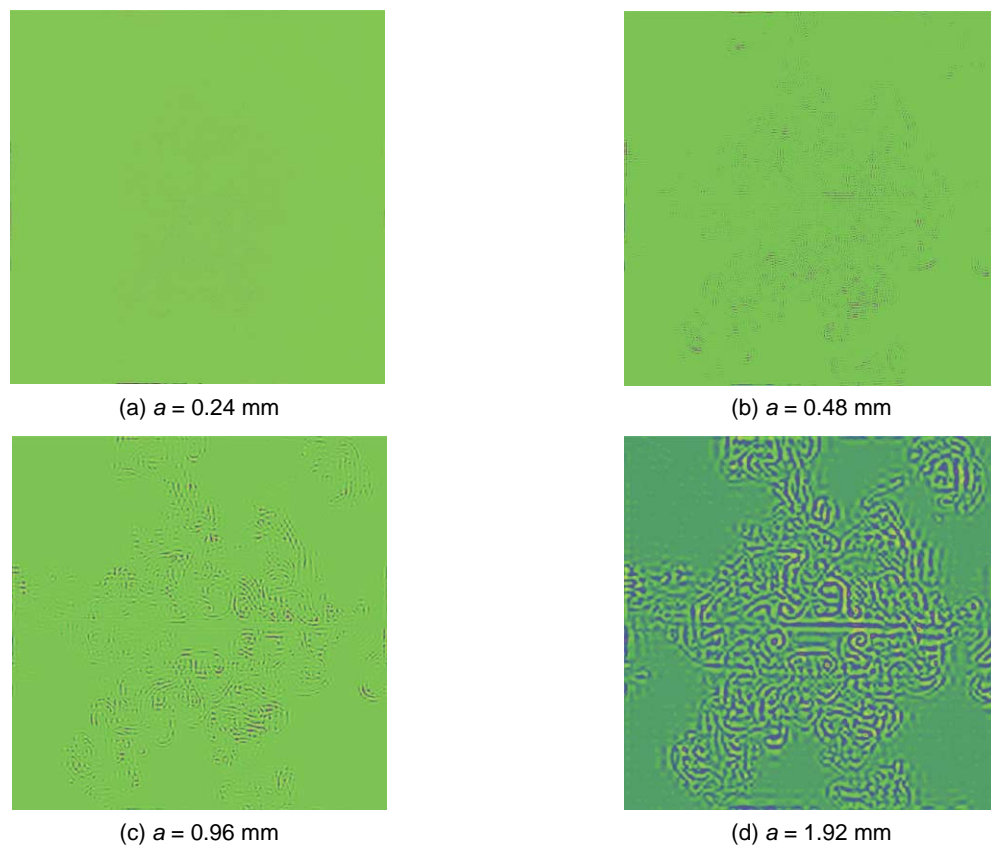


Fig. 7. (continued)

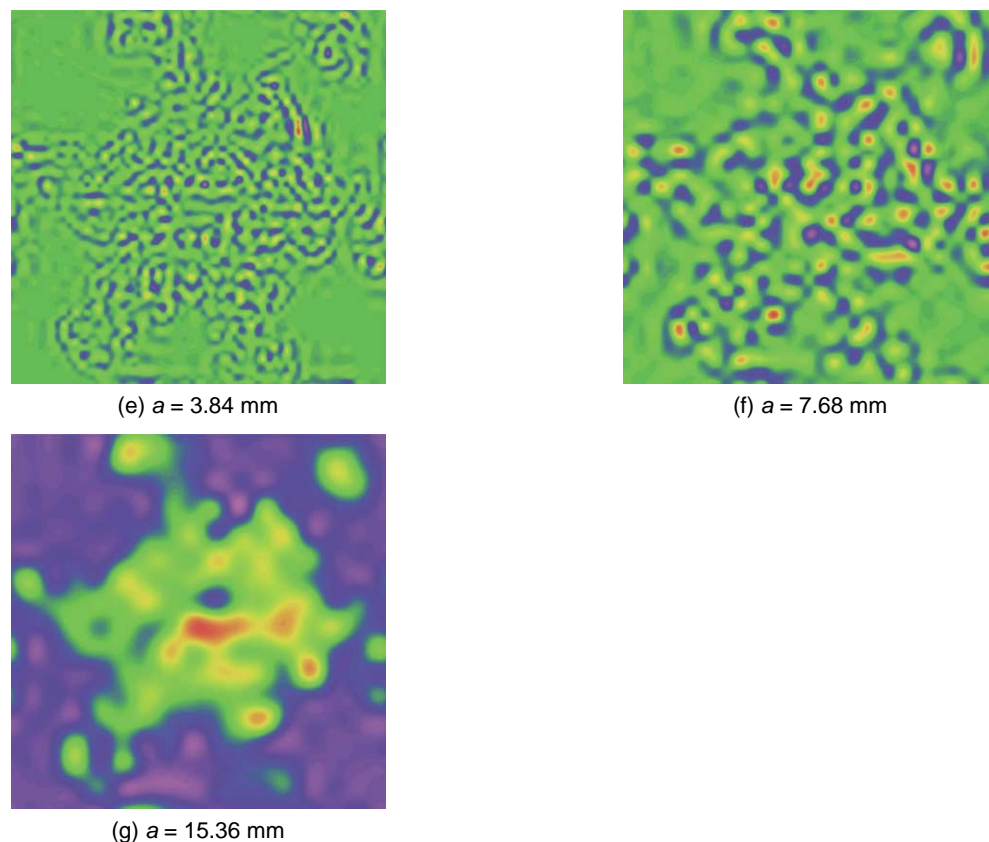


Fig. 7. Multi-resolution images of a lobed mixing jet at  $z/D = 1.5$ .

## 5. Conclusion

In order to visualize the multi-scale turbulent structures of a lobed mixing jet, the LIF cross slice images of flow fields were analyzed by wavelet multi-resolution analysis at two downstream locations of  $z/D = 1.0$  and  $1.5$ . Major conclusions are summarized as follows:

- (1) The multi-resolution images within different broad scales can be obtained using the two-dimensional orthogonal wavelets and provide information on the multi-scale turbulent structures in a lobed mixing jet.
- (2) The large magnitudes of wavelet coefficients of a lobed mixing jet image concentrate on the larger-scale range.
- (3) The core and edge of the vortex can be identified at different broad scales.
- (4) The size of the intermediate- and small-scale structures does not vary significantly with downstream distance.
- (5) The scale range of the coherent structure becomes narrow along the downstream direction.

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