The Effects of Wing Planforms on the Aerodynamic Performance of Thin Finite-Span Flapping Wings

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A three-dimensional high-order unstructured dynamic grid based spectral difference (SD) Navier-Stokes (N-S) compressible flow solver with low Mach number preconditioning is used to investigate the effects of wing planforms on the aerodynamics performance of the thin finite-span flapping wings in this paper. Two types of wings, namely the rectangular and bio-inspired wings, are simulated and compared. The formation process of the ubiquitous two-jet-like wake patterns behind the finite-span flapping wing is explained at first. Then the effects of the wing planforms on the aerodynamics performance of the finite-span flapping wings are elucidated in the terms of the evolutions and dynamic interaction between the leading edge vortices (LEV), trailing edge vortices (TEVs) and the wing tip vortices (TVs) as well as the thrust generated by the flapping wings. Different types of LEVs have been discovered for different wing planforms with the flapping motion in the vertical direction, resulting in different aerodynamic performances. A combined plunging and pitching motion is then adopted to enhance the thrust production of the flapping wing, and it increases the amount of the thrust production by about thirty times when compared with the flapping motion only in the vertical direction.

Nomenclature

\( AoA \) = angle of attack
\( a \) = speed of sound
\( AR \) = aspect ratio
\( C_T \) = thrust coefficient
\( c \) = chord length
\( E \) = total energy
\( F, G, H \) = vectors of fluxes in the physical domain
\( \vec{F}, \vec{G}, \vec{H} \) = vectors of fluxes in the computational domain
\( i,j,k \) = index of coordinates in \( x, y, z \) direction
\( J \) = Jacobian matrix
\( K \) = reduced frequency
\( M \) = Mach number
\( \dot{m} \) = mass flow rate
\( \bar{P} \) = grid displacement
\( p \) = non-dimensional pressure
\( Q, \bar{Q} \) = vectors of conservative variables in the physical and computational domains
\( Re \) = Reynolds number based on the chord length
\( r_5 \) = fifth-order polynomial blending function
\( St \) = Strouhal number
\( s \) = normalized arc length
\( t, \tau \) = time in the physical and computational domain

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u,v,w = non-dimensional velocity in x, y, z direction
\( \vec{v}_g \) = grid velocity
x,y,z = non-dimensional Cartesian coordinates in the physical domain
\( \alpha \) = pitch angle of the airfoil
\( \xi, \eta, \zeta \) = Cartesian coordinates in the computational domain
\( \rho \) = non-dimensional density
\( \phi \) = phase angle
\( \omega_z \) = spanwise (z) vorticity

I. Introduction

Research on the bio-inspired flow under low Reynolds number (10²-10⁴) regimes has increased greatly both in the experiment and computational fluid dynamic (CFD) communities during recent years, encouraged by the design needs of Unmanned Aerial Vehicles (UAVs) and Micro-Air-Vehicles (MAVs). Based on versatile research aims, both two-dimensional (2D) and three-dimensional (3D) unsteady vortex-dominated flows have been studied experimentally and numerically¹. Among all the topics related to the flapping motion, wing/airfoil shape effects have been emphasized by some researchers²-⁴. For the 2D airfoil shape study, Shyy et al.³ studied different airfoil performances for the low Reynolds number and concluded that airfoil shape can dramatically affect the aerodynamic performance. From the research, they found that thin airfoils with relative large cambers can enjoy better aerodynamic performance than their thick counterparts. Lentink and Gerritsma² confirmed that thin airfoils with aft camber perform better under low Reynolds number, compared with traditional thick airfoils. Obviously, airfoils can be treated as a wing with infinite aspect ratio (AR), which cannot reflect the influences of the wing tip vortices over the whole flow field. Therefore, studies of the 3D wing planform effects will help to unfold the 3D wing geometric influences on the vortex interactions, i.e. the wing tip vortices (TVs), leading edge vortices (LEVs) and trailing edge vortices interactions for different wing geometries.

According to the study⁵, wings with low ARs (most insects have such kinds of wings⁶) are beneficial for the agile locomotion. Therefore, many studies¹,³,⁵,⁷-¹¹ have been concentrated on the aerodynamic performances of low AR wings. It can be found that almost all these studies were focused on the unsteady lift generation of the hovering flight. Ellington et al.⁹ first reported that the intense leading edge vortex (LEV) created by dynamic stall is the reason for the high-lift force generation during the hovering flight of the hawkmoth. They also pointed out that the spanwise flow is essential for the stability of such leading edge vortex. However, Birch and Dickinson⁷ found that even there does not exist the spanwise flow, the LEV can still be stabilized for the flapping wings of the fruit fly. They confirmed that the downwash effect of the wing tip vortex can explain the stability of the LEV. In fact, according to Ref. ¹¹, under the regime of high Reynolds numbers, the spanwise flow can enhance the stability of the LEV, while for lower Reynolds numbers, wing tip vortex and wake vorticity play a vital role in prolonging the attachment of the LEV. Recently, Shyy et al.¹¹ further confirmed that the wing kinematics can greatly affect the wing tip effects on the aerodynamic performance of a low AR wing.

For the cruise flight, von Ellenrieder et al.¹² and Parker et al.¹³ have studied the 3D vortex structures after a finite-span flapping wing with two free ends. Their research was restricted to the effects of aerodynamic parameters and the wing motion kinematics on the wake structures. In Spentzos et al.’s⁴ work, the wing tip effects on the dynamic stall are studied. They confirmed that different wing planforms could have similar flow topology. However, whether these conclusions can be extended generally to flapping wings and how the interaction between the wing tip vortex and the leading/trailing edge vortex affects the thrust/lift generation are still open questions. Here, we focus on the wing tip effects on the vortex flow over a series of low AR (=2.6772) fix-rooted wings with different planforms (namely, rectangular, elliptic, and bionic) during the cruise flight.
Since the phenomena in this study are vortex-dominated flows, high-order methods are needed for the numerical simulations due to their prominent low dissipation features. For the present study, a three-dimensional high-order unstructured dynamic grid based spectral difference (SD) Navier-Stokes (N-S) compressible flow solver was adopted. The spectral difference (SD) method$^{14}$ is a recently developed high-order method to solve compressible flow problems on simplex meshes. Its precursor is the conservative staggered-grid Chebyshev multi-domain method$^{15}$. The general formulation of the SD method was first described in Ref. 14 and applied for computational electromagnetic problems. It is then extended to 2D Euler$^{16}$ and Navier-Stokes equations$^{17,18}$. After that, Sun et al.$^{19}$ implemented the SD method for 3D N-S equations on unstructured hexahedral meshes. Later, a weak instability in the original SD method was found independently by Van den Adeele, et al.$^{20,21}$ and Huynh$^{22}$. Huynh$^{22}$ further found that the use of Legendre-Gauss quadrature points as flux points results in a stable SD method. This was later proved by Jameson$^{23}$ for the one dimensional linear advection equation. Based on Sun et al.’s$^{19}$ work, Yu et al.$^{24}$ further developed a Geometric Conservation Law (GCL) compatible SD method for dynamic grids and used it for bio-inspired flow simulations. Liang et al.$^{25}$ and Ou et al.$^{26}$ have developed dynamic grid based SD method for bio-inspired flow simulation as well.

The paper is further organized as follows. In section 2, the SD method, grid deformation strategy and simulation parameters are briefly introduced. Then numerical results are presented and discussed in section 3. There we explain the formation process of the ubiquitous two-jet-like phenomenon behind the finite-span flapping wing and compare the aerodynamic performances of flapping wings with different wing planforms and kinematics. Finally, conclusions are summarized in section 4.

II. Numerical Methods and Simulation Parameters

A. Governing Equations

A three-dimensional high-order Navier-Stokes (N-S) spectral difference solver for dynamic grids has been used for the present simulation. A brief introduction of the method will be presented here and readers can refer to Ref. 24 for details.

We consider the unsteady compressible Navier-Stokes (N-S) equations in conservation form in the physical domain \( (t, x, y, z) \)

\[
\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = 0. \tag{1}
\]

Herein, \( Q = (\rho, \rho u, \rho v, \rho w, E)^T \) are the conservative variables, \( \rho \) is the fluid density, \( u, v \) and \( w \) are the Cartesian velocity components, and \( E \) is the total initial energy. \( F, G, H \) are the total fluxes including both the inviscid and viscous flux vectors, i.e., \( F = F^i - F^v \), \( G = G^i - G^v \) and \( H = H^i - H^v \). Detailed formulas for the fluxes can be found in Ref. 19. On assuming that the fluid obeys the perfect gas law, the pressure is related to the total initial energy by \( E = \frac{p}{\gamma - 1} + \frac{1}{2} \rho (u^2 + v^2 + w^2) \), which closes the solution system.

To achieve an efficient implementation, a time-dependent coordinate transformation from the physical domain \( (t, x, y, z) \) to the computational domain \( (\tau, \xi, \eta, \zeta) \), as shown in Fig. 1(a), is applied on Eq. (1), which is

\[
\frac{\partial \dot{Q}}{\partial \tau} + \frac{\partial \dot{F}}{\partial \xi} + \frac{\partial \dot{G}}{\partial \eta} + \frac{\partial \dot{H}}{\partial \zeta} = 0, \tag{2}
\]

where
Herein, \( \tau = t \), and \( (\xi, \eta, \zeta) \in [-1,1]^3 \), are the local coordinates in the computational domain. In the transformation shown above, the Jacobian matrix \( J \) takes the following form

\[
J = \frac{\partial (x, y, z, t)}{\partial (\xi, \eta, \zeta, \tau)} = \begin{pmatrix}
x_\xi & x_\eta & x_\zeta & x_t \\
y_\xi & y_\eta & y_\zeta & y_t \\
z_\xi & z_\eta & z_\zeta & z_t \\
0 & 0 & 0 & 1
\end{pmatrix}.
\] (4)

For a non-singular transformation, its inverse transformation must also exist, and the transformation matrix is

\[
J^{-1} = \frac{\partial (\xi, \eta, \zeta, \tau)}{\partial (x, y, z, t)} = \begin{pmatrix}
\xi_x & \xi_y & \xi_z & \xi_t \\
\eta_x & \eta_y & \eta_z & \eta_t \\
\zeta_x & \zeta_y & \zeta_z & \zeta_t \\
0 & 0 & 0 & 1
\end{pmatrix}.
\] (5)

It should be noted that all the information concerning grid velocity \( \vec{v}_g = (x_t, y_t, z_t) \) is related with \( (\xi_t, \eta_t, \zeta_t) \) by

\[
\begin{align*}
\xi_t &= -\vec{v}_g \cdot \nabla \xi \\
\eta_t &= -\vec{v}_g \cdot \nabla \eta \\
\zeta_t &= -\vec{v}_g \cdot \nabla \zeta
\end{align*}
\] (6)

**B. Space Discretization**

The SD method is used for the space discretization. In the SD method, two sets of points are given, namely the solution and flux points, as shown in Fig. 1(b). Conservative variables are defined at the solution points (SPs), and then interpolated to flux points to calculate local fluxes. In the present study, the solution points are chosen as the Chebyshev-Gauss quadrature points. It has been proved in Ref. 23 that the adoption of the Legendre-Gauss quadrature points as the flux points can ensure the stability of the SD method. Therefore, the flux points are selected to be the Legendre-Gauss points with both end points as -1 and 1.

Then on using Lagrange polynomials we reconstruct all the fluxes at the flux points. It should be pointed out that this reconstruction is only continuous within a standard element, but discontinuous on the cell interfaces. Therefore, for the inviscid flux, a Riemann solver is necessary to reconstruct a common flux on the interface. Furthermore, since the flow regime for flapping flight is almost incompressible and the present governing equations are compressible Navier-Stokes equations, the Riemann solver should provide good performance at low Mach numbers. The AUSM+-up Riemann solver\(^{27}\) for all speed is selected for the present simulation and is proved to behave well at low Mach numbers. The procedure to reconstruct the common fluxes for the AUSM+-up Riemann solver can be specified as follows.

Suppose the face normal of arbitrary interface denotes as \( \vec{n} \), then the interface mass flow rate \( \dot{m}_{1/2} \) reads

\[
\dot{m}_{1/2} = a_{1/2} M_{1/2} \begin{cases} 
\rho_L & \text{if } M_{1/2} > 0 \\
0 & \text{otherwise},
\end{cases}
\] (7)

where the subscript ‘1/2’ stands for the interface, \( a \) and \( M \) are speed of sound and Mach number respectively. It should be noted that the grid velocity has been included in the interface Mach number \( M \). The numerical normal fluxes \( F^i, G^i \) and \( H^i \) can then be specified as
Figure 1. (a) Transformation from a moving physical domain to a fixed computational domain; (b) Distribution of solution points (circles) and flux points (squares) in a standard quadrilateral element for a third-order accurate SD scheme.

\[
  \begin{aligned}
  \tilde{F}^i &= \left( \tilde{m}_{1/2} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \right. \\
  \tilde{G}^i &= \left( \tilde{m}_{1/2} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \right. \\
  \tilde{H}^i &= \left( \tilde{m}_{1/2} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \right. \\
  &\text{if } \tilde{m}_{1/2} > 0 \\
  &\text{otherwise} + P_{1/2} \end{aligned} \right) |\n|\n\cdot |\n\n\cdot \text{sign}(\hat{n} \cdot \nabla \xi) \tag{8}
\]

where $\psi = \begin{pmatrix} 1, u, v, w, (E + p)/\rho \end{pmatrix}^T$, $P = \begin{pmatrix} 0, p_{nx}, p_{ny}, p_{nz}, 0 \end{pmatrix}^T$, with $n_x, n_y$, and $n_z$ specifying the face normal components in $x$, $y$, and $z$ directions. The superscript ‘i’ indicates the inviscid flux. The reconstruction of the viscous flux is based on a simple average of the ‘left’ and ‘right’ fluxes. The detailed reconstruction procedures are well stated in Ref. 19.

C. Dynamic Grids Strategy

In order to solve problems with moving grids, it is necessary to design a grid moving algorithm. In this study, a blending function approach proposed in Ref. 28 is used to reconstruct the whole physical domain. The fifth-order polynomial blending function reads

\[
  r^i_s(s) = 10s^3 - 15s^4 + 6s^5, s \in [0,1] \tag{9}
\]

It is obvious that $r^i_0(0) = 0$, $r^i_1(1) = 0$, which can generate a smooth variation at both end points during the mesh reconstruction. Herein, ‘s’ is a normalized arc length, which reflects the ‘distance’ between the present node and the moving boundaries. $s=0$ means that the present node will move with the moving boundary, while $s=1$ means that the present node will not move. Therefore, for any motion (transition, rotation), the change of the position vector $\tilde{P}$ is

\[
  \Delta \tilde{P}_{\text{present}} = (1 - r^i_s)\Delta \tilde{P}_{\text{rigid}} \tag{10}
\]

After these manipulations, a new set of mesh nodes can be calculated based on $\Delta \tilde{P}$. It is clear that for systems with complex relative motions, the algebraic algorithm for the grid motion will be hard to design. However, for many cases this method enjoys its remarkable simplicity and efficiency.

D. Numerical Simulations Setup

Rectangular and bio-inspired flapping wings, as shown in Fig. 2 are studies here. Wing surface grids and streamwise grids in the symmetric plane for the simulations are also displayed in Fig. 2. The grid deformation strategy is specified as follows. Suppose that all Lagrangian control points on the flapping wing merely oscillate on the plane perpendicular to the spanwise axis. The maximum position of the profile in the plane perpendicular to the chordwise axis is set to be a parabola with its zero point at the root of the wing, and maximum point at the
wing tip. The rigid-body plunging function for one particular position \((x_0, y_0, z_0)\) on the flapping wing is as follows,

\[
x = x_0, z = z_0, y = y_0 + h_\alpha \sin(\omega t)
\] (11)

where \(h_\alpha\) is determined from the aforementioned parabola. Then the blending function (8) and Eqn. (9) are used to determine the movement of other grid points. Herein, \(\Delta h_{\text{rigid}}\) is specified as \(\Delta y\).

For the combined pitching and plunging motion, the pitching part is controlled as

\[
\begin{align*}
    x_{\text{present}} - x_c &= (x_{\text{former}} - x_c) \cos(\Delta \alpha) - (y_{\text{former}} - y_c) \sin(\Delta \alpha) \\
    y_{\text{present}} - y_c &= (x_{\text{former}} - x_c) \sin(\Delta \alpha) - (y_{\text{former}} - y_c) \cos(\Delta \alpha)
\end{align*}
\] (12)

with \(y_c = h_c \sin(\omega t)\) and \(\alpha = \alpha_0 \cos(\omega t + \phi_0)\). According to the optimal thrust generation conditions suggested by Anderson et al.\(^{29}\), \(\phi_0\) is set as 75°. Herein, \(\Delta h_{\text{rigid}}\) is specified as \(\Delta (x, y)\).

The studied finite-span flapping wings have the same wing span, aspect ratio of the planform and the characteristics of the flapping motion (i.e., sinusoidal trajectory of the flapping wing tip, Strouhal number and reduced frequency). In the present study, the Strouhal number (\(St\)) of the finite-span flapping wings was selected to be well within the optimal range usually used by flying insects and birds and swimming fishes (i.e., \(0.2 < St < 0.4\)). For all the simulations during the present study, the Mach number of the free stream is set to be 0.05, under which the flow is almost incompressible. The APs for all wings are set as 2.6772. The Reynolds number (\(Re\)) based on the free stream velocity and the maximum chord length is 1200. The reduced frequency (\(K\)) of the flapping motion is 3.5, and the Strouhal number of the wingtip, based on the definition in Ref. 30, is 0.38. All these parameters are from the experimental setup stated in Ref. 10. The space discretization accuracy for the simulation is of third order, and the time integration is performed with the explicit third order TVD Runge-Kutta method.

III. Numerical Results and Discussions

The comparisons of the instantaneous vorticity distributions from the numerical simulations and those from experimental measurements in the chordwise cross plane at 50%, 75% and 100% wingspan (i.e., wingtip) and the corresponding time-averaged velocity fields are displayed in Fig. 3. Note that the wake structures at 50% wingspan from the numerical simulations bare good visual agreement with the experimental results at the same position. However, at 75% wingspan and the wingtip, numerical results exhibit more elaborate small vortices structures than the experimental results. From the corresponding time-averaged velocity fields at all three positions as displayed in Fig. 3, it is found that the numerical simulations capture the qualitative features of the
Figure 3. Instantaneous vorticity fields and the corresponding time-averaged velocity fields at (a) 50%, (b) 75% wingspan and (c) wingtip for the flapping rectangular wing. Left two columns: numerical results; Right two columns: experimental results (Courtesy of H. Hu, et al.\textsuperscript{10}).

Wakes indicated by experimental measurements. Specifically, the experimental measurements show a von Karman vortex street type wake at 50% wingspan, which is well captured by the numerical simulations. Although numerical simulations show more elaborate vortices structures at 75% wingspan and the wingtip, the time-averaged results show the similar flow topology as revealed by the experimental results --- jet-like flow structures, which indicates the momentum surplus. Furthermore, these jet-like flow structures are not formed merely by a single inverse von Karman vortex street, but the combination of several inverse von Karman vortex streets (wingtip) or even the mixture of both regular and inverse ones (75% wingspan), which has been reported as bifurcated jets wake pattern in Dong et al.'s\textsuperscript{8} study for a free-end finite-span wing. These interesting wake phenomena are considered to be closely tied to the wingtip vortices effects, which will be thoroughly discussed next.

A. Two-Jet-Like Wake Patterns Formation

The wake vortex structures of the flapping rectangular wings from perspective and side views are shown in Fig. 4(a) and (b) respectively. In these figures, the vortex structures are indicated by the Q-criterion colored with the streamwise velocity. The Q-criterion is a Galilean-invariant vortex criterion, which is defined as follows
where $R_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$, is the angular rotation tensor, and $S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, is the rate-of-strain tensor. The different vortices have been marked out with rectangular windows or solid arrows which indicate the rotation directions. It is clear from the figures that the complex vortex system around the flapping wing can be decomposed into four parts, namely trailing edge vortices (TEVs), leading edge vortices (LEVs) and tip vortices (TVs), and the entangled vortices (EVs) due to the interactions among TEVs, TVs and LEVs. Similar wake phenomena have been reported by Dong, et al. for free-end finite-span wings except the complex EVs. This might be due to the fact that in the present study a thin wing with shape edges are used for the simulations while in the aforementioned literatures, relatively thick wings are adopted in the simulations.

It has been reported in Ref. 8 that the formation of the two-jet-like wake patterns behind the flapping wing is closely related to the existence of tip vortices. But the reasons for the formation process of the bifurcated jet were not fully analyzed. Herein, a detailed observation of the bifurcated jet effects is shown in Fig. 4(c) for the fixed-root flapping rectangular wing. The figure shows the 3D vorticity fields indicated by the Q-criterion and the spanwise vorticity field at the 75% wingspan. The trajectories of both clockwise (-) and anti-clockwise (+) vortices are also schematically plotted in the figure. Furthermore, the jet bifurcation position is determined by examining the starting point of the two-jet-like wake patterns from the time-averaged velocity fields in Fig. 3. It is observed from the figure that the jet bifurcation occurs when TVs intensively interact with the TEVs and many elaborate small vortices appear in this region.

In order to further examine the physics behind this, a combined flapping and pitching motion with pitch leading plunge 75° is used to reduce the separation from the leading edge and the wingtip, which makes the wake vortex structures much clearer as shown in Fig. 5(a). Note that with this combined motion, the two-jet-like patterns still exists at the 75% wingspan as shown in Fig. 6. From Fig. 5(b), it is obvious that the upper branch of the bifurcated jet is formed by an anti-clockwise vortex row consisting of TEVs and a clockwise vortex row consisting of TVs; while the lower branch of the bifurcated jet is formed by an anti-clockwise vortex row consisting of TVs and a clockwise vortex row consisting of TEVs. The reasons why TVs can contribute to the spanwise vorticity are illustrated in Fig. 5(b). Let’s use TVs2, TEVs3 and TVs3 to explain the process. Because of the existence of TVs2, the end part of the TEVs near the wingtip will be dragged gradually from the ‘z’ direction to the ‘y’ direction during the flapping stroke, which indicates that certain amount of vorticity in the vertical (y) direction is generated. The induced rotational velocity field is schematically denoted with the blue dashed arrow near the wingtip part of TEVs3. This velocity field will bend the bottom end of TVs3 to TEVs2, and finally TVs3 have a vorticity component in the spanwise direction. It is not hard to examine that this induced vorticity component is negative as denoted with the blue dashed arrow near the bottom part of TVs3. This explains the formation of the spanwise vorticity contribution from the TVs and further elucidates the formation of the two-jet-like wake patterns. Note that the above explanation will also work for the flapping case aforementioned, although the existence of small vortices in that case makes the two-jet-like wake formation process hard to distinguish. Similar explanations can apply to the formation of the wake pattern at the wingtip.
Figure 4. Vortex topology around the flapping rectangular wing. Vortex structures are indicated by the Q-criterion and colored by the streamwise velocity. (a) Perspective view. (b) Side view. (c) Perspective view of the vortex structures near the wingtip region and the spanwise vorticity field of the chordwise cross plane at 75% wingspan.
Figure 5. Vortex topology around the rectangular wing with a combined plunging and pitching motion. Vortex structures are indicated by the Q-criterion and colored by the streamwise velocity. (a) Side view. (b) Perspective view of the vortex structures near the wingtip region and the spanwise vorticity field of the chordwise cross plane at 75% wingspan.

Figure 6. Time-averaged velocity fields at (a) 50%, (b) 75% and (c) 100% wingspan for the rectangular wing with a combined plunging and pitching motion.

B. Aerodynamic Performance Comparisons of Different Wing Planforms

The comparison of thrust coefficient histories for the rectangular and bio-inspired wings with the flapping motion is displayed in Fig. 7(a). The contributions from the pressure force and viscous force for the thrust are shown in Fig. 7(b) and (c). From these figures, it is found that during one flapping cycle the rectangular wing experiences both larger thrust and drag than the bio-inspired wing. These differences mainly come from the contributions from the pressure force. It is also observed that the bio-inspired wing experience less drag from the viscous force (i.e., shear stress). The time-averaged thrust coefficients for these two wings as presented in Table 1. From the table, it is clear that the rectangular wing generates larger thrust than the bio-inspired wing, and for both wings, the pressure force dominates the thrust production. Note that both thrust coefficient histories for the two wings in Fig. 7 display certain small-scale unsteady fluctuations. This can be explained by the rich vortex structures around the flapping wings as shown in Fig. 8. In this figure, the vortex structures indicated by the Q-criterion and colored by the streamwise velocity for the rectangular wing are shown in the upper row. The phases for the four instantaneous flow fields are 0°, 90°, 180°, and 270° respectively. The corresponding flow fields at the same phases for the bio-inspired wing with the same Q value are displayed in the lower row of Fig. 8. It is found that more small vortex structures are generated around the rectangular wing especially in the wingtip region. This is due to the relatively sharp tip edge of the rectangular wing and this phenomenon indicates that more flapping energy has been wasted as these small vortices are hard to be efficiently collected to generate thrust. It is also found that the LEVs around the rectangular wing at phases 0° and 180° are stronger than those around the bio-inspired wing. As will be discussed momentarily, at these two phases the flapping wings will experience thrust peaks, which are believed to closely tie to the behaviors of the LEVs. Further, this phenomenon can help to explain the reason why the rectangular wing generates more thrust than the bio-inspired wing does at the present simulation parameters.

As aforementioned, the pressure force dominates the thrust generation in the present cases. Based on careful examinations of the flow fields it is found that the pressure change on the leading and trailing edges of the flapping wings mainly occurs in the regions near the wingtip, indicating that the thrust generation is dominated by the outer 50% regions of the flapping wings. It is obvious that at these regions flapping wings have larger flapping amplitudes and speeds and can result in a local dynamic stall. The associated LEVs can be stabilized by
the downwash effects of the TVs and can induce a local low pressure region near the leading edge of the flapping wing. This is beneficial for the thrust production. Since the LEVs of the rectangular wing are more intensively interacting with the leading edge than the bio-inspired wing, a lower pressure region near the wingtip of the rectangular wing makes it experience a larger thrust peak than the bio-inspired wing as observed from the thrust coefficient histories (Fig. 7).

The spanwise vorticity fields and the time-averaged velocity fields of the bio-inspired wing in the chordwise cross plane at 50%, 75% wingspan and wingtip are displayed in Fig. 9. The phases of the flow fields and contour levels are the same as those shown in Fig. 3 for the rectangular wing. Different features of the flow fields can be found by comparing Fig. 9 and 3. At the 50% wingspan, it is clear that the bio-inspired wing has smaller region with velocity deficit than the rectangular wing does. However, at the wingtip, because of the wing shape difference the bio-inspired wing almost does not generate thrust while the rectangular wing still has apparent jet patterns. At the 75% wingspan, the two-jet-like pattern for the bio-inspired wing has a different shape compared with the rectangular wing, and the high speed region is smaller than the rectangular wing. Since it has been found that the thrust production is mainly determined by the outer 50% regions of the flapping wing, all the discussed wake features for the rectangular and bio-inspired wings show the reasons why the rectangular wing generates more thrust than the bio-inspired wing from the aspect of wake structures.

C. Aerodynamic Performances for Wings with Combined Plunging and Pitching Motions

Note that according to Table 1 the time-averaged thrust coefficient for the flapping rectangular wing is very small. This can be explained like this. Based on the knowledge that the pressure force dominates the thrust generation, two parameters, namely the effective wing area projection in the streamwise direction and the pressure difference, will determine the output of the thrust during the flapping flight. Since thin wings are adopted in the present study, for the flapping motion in the vertical direction, the wing area projection in the streamwise direction is very small. This is unfavorable to the thrust production. Therefore, it becomes necessary to add certain pitching motion to the flapping motion to enlarge the wing area projection in the streamwise direction. However, the phase lag between the plunging motion and the pitching motion should be carefully designed as this phase lag will affect the adjustment of effective angle of attack (AoA). If this parameter is not assigned properly, the performance of the wing can even degrade. As aforementioned, the phase lag between the plunging motion and the pitching motion is set to be 75° as suggested by Anderson et al.29.

The histories of the total thrust coefficient and the component contributed by the pressure force for the rectangular wing under combined plunging and pitching motions are displayed in Fig. 10. The corresponding vortex structures indicated by the Q-criterion around the wing are shown in Fig. 11 for four phases, namely 0°, 90°, 180° and 270°. It is concluded that under the combined motions, the flapping wing can generate much larger (about thirty times) thrust than the flapping case. Moreover, because of the effective AoA adjustment of the pitching motion, the breakdown of vortices becomes less severe, which indicates less energy waste.

IV. Conclusions

In this study, a three-dimensional high-order unstructured dynamic grid based spectral difference (SD) Navier-Stokes (N-S) flow solver was used to investigate the effects of wing planforms on the aerodynamics performance of the thin finite-span flapping wings. The formation process of the two-jet-like wake patterns behind the finite-span flapping wing is found to be closely related to the interaction between TEVs and TVs. It is found that for the flapping motion in the vertical direction, the rectangular wing can generate larger thrust than the bio-inspired wing because of the dynamic behaviors of LEVs. However, the thrust production for such kind of motions is very small. In order to enhance the thrust production, a combined plunging and pitching
motion is adopted and it increases the amount of the thrust production by about thirty times when compared with the flapping motion only in the vertical direction.

References


Figure 7. The thrust coefficient histories for different wing planforms with the flapping motion. (a) total thrust; (b) contribution from the pressure force; (c) contribution from the viscous force.
Table 1. Time-averaged thrust coefficient histories for different wing planforms with the flapping motion or the combined motion indicated by ‘Com.’. $\bar{C}_T$ stands for the time-averaged total thrust; $\bar{C}_{T,P}$ stands for the contribution from the pressure force; $\bar{C}_{T,V}$ stands for the contribution from the viscous force.

<table>
<thead>
<tr>
<th>Planform</th>
<th>$\bar{C}_T$</th>
<th>$\bar{C}_{T,P}$</th>
<th>$\bar{C}_{T,V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>$1.36 \times 10^{-2}$</td>
<td>$4.72 \times 10^{-2}$</td>
<td>$-3.36 \times 10^{-2}$</td>
</tr>
<tr>
<td>Bio-inspired</td>
<td>$0.17 \times 10^{-2}$</td>
<td>$3.07 \times 10^{-2}$</td>
<td>$-2.90 \times 10^{-2}$</td>
</tr>
<tr>
<td>Rectangular (Com.)</td>
<td>0.366</td>
<td>0.466</td>
<td>0.100</td>
</tr>
</tbody>
</table>

Figure 8. Comparison of the vortex topology for the rectangular and bio-inspired wings at four phases ($0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$) with the flapping motion. The upper row is for the rectangular wing while the lower row is for the bio-inspired wing.
Figure 9. Instantaneous vorticity fields and the corresponding time-averaged velocity fields at (a) 50%, (b) 75% wingspan and (c) wingtip for the flapping bio-inspired wing.

Figure 10. The total thrust coefficient history and that from the pressure force for the rectangular wing with the combined motion.

Figure 11. Vortex topology for the rectangular at four phases (0°, 90°, 180° and 270°) with the combined motion.