Experimental and Numerical Investigations on the Asymmetric Wake

Vortex Structures around an Oscillating Airfoil

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Wake structures behind a sinusoidally pitching NACA0012 have been studied with both experimental and numerical approaches. The results from Particle Image Velocimetry (PIV) measurements and those from the high-order unstructured dynamic grid based spectral difference (SD) compressible Navier-Stokes (NS) solver agree well with each other. Two types of wake transition processes, namely the transition from a drag-indicative wake to a thrust-indicative wake and that from the symmetric wake to the asymmetric wake are experimentally investigated. Asymmetric wake phenomena have been emphasized in the study. The deflected wake is found to appear at approximately Strouhal number 0.31 and reduced frequency 15.1 for the pitching amplitude 5°. As the Strouhal number increases, the dipole mode of the vortex pair becomes more apparent, which is considered to be a vital element to form the asymmetric wake. Besides, the dependency of the deflective directions of the asymmetric wake on other parameters, e.g. initial phase angle, reduced frequency, has been analyzed both experimentally and numerically.

Nomenclature

\begin{itemize}
\item \textit{AoA} = angle of attack
\item \textit{a} = speed of sound
\item \textit{C_L} = lift coefficient
\item \textit{C_T} = thrust coefficient
\item \textit{c} = chord length
\item \textit{E} = total energy
\item \textit{F}, \textit{G} = vectors of fluxes in the physical domain
\item \textit{\bar{F}}, \textit{\bar{G}} = vectors of fluxes in the computational domain
\item \textit{i,j} = index of coordinates in \textit{x}, \textit{y} direction
\item \textit{J} = Jacobian matrix
\item \textit{K} = reduced frequency
\item \textit{M} = Mach number
\item \textit{\dot{m}} = mass flow rate
\item \textit{p} = non-dimensional pressure
\item \textit{\bar{Q}}, \textit{\bar{\bar{Q}}} = vectors of conservative variables in the physical and computational domains
\item \textit{Re} = Reynolds number based on the chord length
\item \textit{r_5} = fifth-order polynomial blending function
\item \textit{St} = Strouhal number
\item \textit{s} = normalized arc length
\item \textit{t}, \textit{\tau} = time in the physical and computational domain
\item \textit{u,v} = non-dimensional velocity in \textit{x}, \textit{y} direction
\item \textit{\bar{v}} \textit{a} = grid velocity
\item \textit{x,y} = non-dimensional Cartesian coordinates in the physical domain
\end{itemize}

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IRD-SIZED and insect-sized Micro-Air-Vehicles (MAVs) have opened up new opportunities for surveillance missions. Their miniaturized size and versatile functionality offers a significant advantage over conventional approaches in high-density or bio-harmful environments. MAVs can be designed with fixed, rotary or flapping wings. Among them, the flapping wing motion, which has been widely used by natural flyers, is attractive due to its high efficiency and excellent maneuverability. It is observed that in natural flights within the Reynolds number range of $10^3$ to $10^5$, a thrust can be generated from flapping motions due to the generation of the reverse von Karman vortex street behind the flapping wing. However, it is challenging to fully understand the aerodynamics exhibited by natural flapping wing flyers considering the intricacies behind them. Therefore, it is reasonable to simplify the complicated system by substituting it with decent models of different modeling levels. By studying these simplified models, one can subtract the underlying physics behind the bio-inspired unsteady flow piece by piece and in turn, to apply them in the design of MAVs. Readers who are interested in these topics can refer to two comprehensive reviews by Shyy et al.\textsuperscript{1,2}.

Among all research directions, wake structure analyses have attracted much attention, as the evolution of the vortex street reveals the flapping wing dynamics. Myriads of experiments and numerical simulations have been carried out for both two dimensional (2D) and three dimensional (3D) wake structure analyses. Koochesfahani\textsuperscript{3} studied the wake patterns for a pitching NACA0012 airfoil with the small pitching amplitude but relatively large pitching frequency through the dye visualization and laser Doppler velocimetry (LDV) measurement. From that study, they concluded that the wake pattern can be controlled by adjusting the frequency, amplitude and the shape of the oscillation wave form. In a successive work, Bohl and Koochesfahani\textsuperscript{4} studied the flow over a sinusoidally pitching NACA0012 airfoil of various reduced frequencies using molecular tagging velocimetry (MTV). In that research, they reported the transition point for a von Karman vortex street to a reverse von Karman vortex street for the pitching motion with small pitching amplitude ($2^\circ$) but relatively large reduced frequency. Through the linear stability analyses of the wake structures, Triantafyllou et al.\textsuperscript{5} obtained the optimal wake width based Strouhal number for oscillating foils in the range 0.25 to 0.35. Anderson et al.\textsuperscript{6} experimentally confirmed that the propulsion efficiency at the optimal Strouhal number 0.3-0.4 can be up to 87% when the principle mechanism governing the wake vortex dynamics is well considered. Based on the numerical studies of a heaving airfoil, Wang\textsuperscript{7} also obtained that the optimal Strouhal number is around 0.3 and showed certain criteria to achieve the optimal performance. Lai and Platzer\textsuperscript{8} experimentally studied the wake features after a plunging NACA0012 airfoil, and confirmed that a plunging foil could generate a jet-like wake structure. Further, based on the dye visualization, they showed schematically the processes for the generations of different wake patterns. Later, extensive numerical simulations have been performed by Young and Lai\textsuperscript{9}, focusing on the relationships between flow features and the aerodynamic parameters. Godoy-Diana et al.\textsuperscript{10} experimentally studied the aerodynamic parameter dependency of the transition process from the drag-generation wake to the thrust-generation one for a teardrop like pitching foil based on the wake visualization and displayed the parameter map for different wake types. Compared with 2D wake structures, 3D wake patterns are more complex and fewer results are reported on this topic. Ellenrieder et al.\textsuperscript{11} analyzed the Strouhal number and plunge/pitch phase lag effects on the wake vortex structures behind

\begin{align*}
\alpha &= \text{pitch angle of the airfoil} \\
\xi, \eta &= \text{Cartesian coordinates in the computational domain} \\
\rho &= \text{non-dimensional density} \\
\phi &= \text{phase angle} \\
\omega_z &= \text{spanwise (z) vorticity}
\end{align*}
investigating the wake visualization results from experiments and numerical simulations, researchers found an interesting asymmetric wake vortex shedding mode which deserves to be further studied. the key problem originated from the asymmetric wake phenomenon can be stated like this. intuitively, if an axisymmetric airfoil oscillates around the balancing position of zero angle of attack (aoa) using a symmetric motion algorithm, e.g. sinusoidal pitch or plunge, the wake direction should always follow the stream wise direction. this is true under the small strouhal number as has been confirmed by many experimental and numerical results. however, it is found that as the strouhal number increases, a deflected wake can be observed behind an oscillating foil. it is an open question to answer the originality of the asymmetric wake. furthermore, since the deflected wake is usually accompanied with larger thrust and lift generation, it seems that to fully understand the characteristics of this phenomenon might be vital for the design and control of mavs. as reported by jones et al., a deflected wake occurs generally with a relative larger wake width based strouhal number (around 0.48). beside, during the experiment, they found that the direction of the deflected wake pattern can alter somewhat arbitrarily due to some reasons that were not clear. through a numerical simulation with an inviscid panel code, the asymmetric wake can be predicted in their research. platzer et al. suggested that for flow with large strouhal numbers, a viscous flow solver is necessary to provide a complete picture of the occurrence of the deflected wake structure. lewin and haj-hariri used an incompressible viscous flow solver to study the aerodynamic parameter dependency of thrust generation for a heaving airfoil. from their research, wake patterns are detailed categorized, and deflected wakes those can alter directions are reported. the possible reason can be contributed to the intensive interactions between leading edge vortices (levs) and trailing edge vortices (tevs). heathcote and gursul found that the period of the jet switching during their experiment is two orders of magnitude greater than the heave period of the foil. they also showed that the jet switching period tightly depends on the strouhal number and the stiffness of the airfoil. godoy-diana et al. showed the watershed for the symmetric reverse von karman vortex street and the asymmetric jet-like wake in a frequency-amplitude map. from this diagram, the authors concluded that asymmetric wake patterns can even fall into the optimal strouhal number region. later, godoy-diana et al. modeled the asymmetric wake phenomenon by using a dipole model, which has been termed as a dual mode in jone et al.'s work. a similar approach has been used to model the asymmetric wake for a semi-infinite vortex array by yu et al. ellenrieder and pothos experimentally verified the onset of the deflected wake and the corresponding flow features. they claimed that when strouhal number exceeds 0.434, a deflected wake will appear for a plunging naca0012 airfoil. liang et al. numerically found that the deflective angle decreases when the reynolds number decreases for a fast plunging airfoil. as suggested by some researchers, the 3d counterpart of the 2d asymmetric wake might be the two jet-like wake patterns behind a free-end finite-span wing, as shown in ref. 12. recently, hu et al. have experimentally verified that for a fixed-root wing, the two jet-like wake patterns can also be discovered. whether these flow structures can be related to the 2d asymmetric wake pattern still needs more investigations.
Obviously, the aforementioned unsteady bio-inspired flows are dominated by moving vortices, which play a critical role in the wake pattern analyses. As 1st and 2nd order flow solvers may dissipate the unsteady vortices, a high-order dynamic grid based spectral difference (SD) method is used in tackling these unsteady vortex-dominated flows. As the high order method means possible high cost, an efficient high-order method needs to be developed for the bio-inspired flow simulations. The SD method used in the present study is one of good choices. It has been demonstrated that the dynamic grid based SD method works well for the bio-inspired flows. The basic approach to achieve high-order accuracy in the SD method is to use a high degree polynomial to approximate the exact solution in a local element (a standard cell). However, not like the discontinuous Galerkin (DG) method and spectral volume (SV) method, the SD method is in the differential form, which is efficient and simple to implement. Furthermore, the SD method can enjoy larger time steps than the DG type method, which might make it more competitive for the potential engineering use. Recently, all kinds of high-order methods have been used for the bio-inspired flows. Visbal has successfully utilized a high-order method (a compact finite-difference approach) to simulate the flow field around a SD7003 airfoil. Persson et al. have developed a dynamic DG method for a finite-span wing simulation and compared the results with other numerical methods. Liang et al. have successfully used SD method for a plunging NACA0012 airfoil simulation. In their research, the asymmetric wake has been reported under large Strouhal number and the relationship between the deflected direction and Reynolds number has been studied. Several potential applications for SD method in the bio-inspired flow have been reported by Yu et al. They also discovered that the asymmetric wake phenomena can appear under Strouhal number 0.33 with no leading edge separation. Ou et al. developed a 3D SD solver for the finite-span flapping wing simulations, and verified the effectiveness of the method for the bio-inspired flow simulations.

Based on the above discussions, the present paper endeavors to further study the conditions for the occurrence of the deflected wake and how these conditions affect the dynamic behaviors of the deflected wake. The remaining paper is organized as follows. In section 2 and 3, the experimental setup and the numerical method will be introduced. The numerical simulation setup is also specified in section 3. Then experimental and numerical results are displayed in section 4. Two wake transition processes are discussed there and the asymmetric wake phenomenon is analyzed through both experimental and numerical approaches. Finally, conclusions are summarized in section 5.

II. Experimental setup

The experimental study was performed in a closed-circuit low-speed wind tunnel located in the Aerospace Engineering Department of Iowa State University. A glass-walled test section is mounted on this wind tunnel, and the interior dimensions of the test section are of 2.5m \( \times \) 0.3m \( \times \) 0.3m. The tunnel has a 40:1 contraction section upstream of the test section and has honeycombs, screen structures and a cooling system installed ahead of the contract section. A uniform incoming flow with low turbulent intensity can then be provided for the test section. For the present study, the free stream velocity for the test section is maintained at 0.5m/s.

The NACA0012 airfoil used in the present study has a chord length of 4 inch and spanwise length of 11.5 inch. The airfoil model was manufactured with a 3D plastic printer located in the Aerospace Engineering Department of Iowa State University. A linkage mechanism, which is driven by a servo-controlled DC motor, is used to provide the sinusoidal pitching motion \( \alpha = a_0 \cos(\omega t + \phi_0) \) of the airfoil as shown in Fig. 1(a). The experimental setup for the phase-lock PIV measurement is schematically displayed in Fig. 1(b). During the experiment, the flow was seeded with 1-5 micro-meter oil droplets. A
double-pulsed Nd:YAG laser (NewWave Research Solo) adjusted on the second harmonic and emitting two laser pulses at a wavelength of 532 nm at a repetition rate of 2 Hz was used to illuminate the wake region behind the airfoil via the reflection of a mirror mounted under the test section. The laser sheet was created from a laser beam by inserting a set of spherical and cylindrical lenses between the laser and mirror. The laser sheet was positioned near the middle span of the airfoil and had a thickness of approximately 1 mm. A high-resolution 12-bit (1,600 × 1,200 pixel) CCD camera (PCO 1600, CookeCorp) was used for PIV image acquisition. The time sequence for the trigger of laser and camera was controlled by the Digital Delay Generator (Berkeley Nucleonics, Model 565). For the phase-lock measurement, the delay generator was linked to a digital pulse generator to provide a trigger signal. A tachometer is used to capture the phase information and supply the signal to the digital pulse generator.

For the post processing, instantaneous PIV velocity vectors were obtained by using a frame to frame cross-correlation technique involving successive frames of patterns of particle images in an interrogation window of 32 × 32 pixels with an effective overlap of 50% of the interrogation windows. The spanwise vorticity was then calculated from the velocity field.

![Figure 1. Sketch of (a) the experimental setup for the PIV measurement and (b) the linkage system used to generate a sinusoidal pitching motion for the airfoil.](image)

### III. Numerical method

#### A. Governing Equations

Numerical simulations are performed with an unsteady compressible Navier-Stokes solver using dynamic unstructured grid based high-order spectral difference (SD) method developed in Ref. 24. The 2D unsteady compressible Navier-Stokes equations in conservation form read,

\[ \frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0. \]  

(1)

Herein, \( Q = (\rho, \rho u, \rho v, E)^T \) are the conservative variables, and \( F, G \) are the total fluxes including both the inviscid and viscous flux vectors, i.e., \( F = F^i - F^v \) and \( G = G^i - G^v \), which takes the following form.
In (2), \( \rho \) is the fluid density, \( u \) and \( v \) are the Cartesian velocity components, \( p \) is the pressure, and \( E \) is the total energy, \( \mu \) is dynamic viscosity, \( C_{pre} \) is the specific heat at constant pressure, \( P_r \) is the Prandtl number, and \( T \) is the temperature. The stress tensors in (2) take the following form
\[
\begin{align*}
\tau_{xx} &= 2\mu \left( u_x - \frac{u_x + v_y}{3} \right), \\
\tau_{yy} &= 2\mu \left( v_y - \frac{u_x + v_y}{3} \right), \\
\tau_{xy} &= \tau_{yx} = \mu \left( v_x + u_y \right)
\end{align*}
\] (3)

On assuming that the fluid obeys the perfect gas law, the pressure is related to the total initial energy by
\[
E = \frac{p}{\gamma - 1} + \frac{1}{2} \rho (u^2 + v^2)
\]
with a constant ratio of the specific heats \( \gamma \), which closes the solution system.

To achieve an efficient implementation, a time-dependent coordinate transformation from the physical domain \( (t, x, y) \) to the computational domain \( (\tau, \xi, \eta) \), as shown in Fig. 2(a), is applied on Eq. (1), which is
\[
\frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial \tilde{F}}{\partial x} + \frac{\partial \tilde{G}}{\partial y} = 0,
\] (4)
where
\[
\begin{align*}
\tilde{Q} &= |J| \tilde{Q} \\
\tilde{F} &= |J| \left( Q\xi_x + F\xi_x + G\xi_y \right) \\
\tilde{G} &= |J| \left( Q\eta_x + F\eta_x + G\eta_y \right)
\end{align*}
\] (5)

Herein, \( \tau = t \) and \( (\xi, \eta) \in [-1,1]^2 \), are the local coordinates in the computational domain. In the transformation shown above, the Jacobian matrix \( J \) takes the following form
\[
J = \frac{\partial (x,y,t)}{\partial (\xi,\eta,\tau)} = \begin{pmatrix}
x_x & x_\eta & x_t \\
y_x & y_\eta & y_t \\
0 & 0 & 1
\end{pmatrix}.
\] (6)

It should be noted that the grid velocity \( \tilde{v}_g = (x_t,y_t) \) is related with \( (\xi_t,\eta_t) \) by
\[
\begin{align*}
\xi_t &= -\tilde{v}_g \cdot \nabla \xi \\
\eta_t &= -\tilde{v}_g \cdot \nabla \eta
\end{align*}
\] (7)

B.Space Discretization

The SD method is used for the space discretization. In the SD method, two sets of points are given, namely the solution and flux points, as shown in Fig. 2(b). Conservative variables are defined at the solution points (SPs), and then interpolated to flux points to calculate local fluxes. In the present study, the solution points are chosen as the Chebyshev-Gauss quadrature points. A weak instability for the choice of the flux points in the original SD method was found independently by Van den Abeele, et al.\textsuperscript{30,31} and Huynh\textsuperscript{32}. Huynh\textsuperscript{32}\textsuperscript{Error! Reference source not found.} further found that the use of Legendre-Gauss quadrature points as flux points results in a stable SD method. It is then proved mathematically in Ref. 33 that the adoption of the Legendre-
Gauss quadrature points as the flux points can ensure the stability of the SD method. Therefore, the flux points are selected to be the Legendre-Gauss points with both end points as -1 and 1.

Then on using Lagrange polynomials we reconstruct all the fluxes at the flux points. It should be pointed out that this reconstruction is only continuous within a standard element, but discontinuous on the cell interfaces. Therefore, for the inviscid flux, a Riemann solver is necessary to reconstruct a common flux on the interface. For a moving boundary problem, since the eigenvalues of the Euler equations are different from those for a fixed boundary problem by the grid velocity, the design of the Riemann solver should consider the grid velocity. Furthermore, since the flow regime for flapping flight is almost incompressible and the present governing equations are compressible Navier-Stokes equations, the Riemann solver should provide good performance at low Mach numbers. The AUSM+-up Riemann solver for all speed is selected for the present simulation and is proved to behave well at low Mach numbers. The procedure to reconstruct the common fluxes for the AUSM+-up Riemann solver can be specified as follows.

Suppose the face normal of arbitrary interface denotes as \( \vec{n} \), then the interface mass flow rate \( \dot{m}_{1/2} \) reads

\[
\dot{m}_{1/2} = \alpha_{1/2} M_{1/2} \begin{cases} 
\rho_l & \text{if } M_{1/2} > 0 \\
\rho_R & \text{otherwise}
\end{cases}
\]

where the subscript ‘1/2’ stands for the interface, \( \alpha \) and \( M \) are speed of sound and Mach number respectively. It should be noted that the grid velocity has been included in the interface Mach number \( M \).

The numerical normal fluxes \( \mathcal{F}^{i} \) and \( \mathcal{G}^{i} \) can then be specified as

\[
\begin{align*}
\mathcal{F}^{i} &= \left( \begin{array}{c} \psi_L \\ \psi_R \end{array} \right) \begin{cases} 
\dot{m}_{1/2} \psi_L & \text{if } \dot{m}_{1/2} > 0 \\
\dot{m}_{1/2} \psi_R & \text{otherwise}
\end{cases} + P_{1/2} \left| \nabla \xi \right| \text{sign}(\vec{n} \cdot \nabla \xi) \\
\mathcal{G}^{i} &= \left( \begin{array}{c} \psi_L \\ \psi_R \end{array} \right) \begin{cases} 
\dot{m}_{1/2} \psi_L & \text{if } \dot{m}_{1/2} > 0 \\
\dot{m}_{1/2} \psi_R & \text{otherwise}
\end{cases} + P_{1/2} \left| \nabla \eta \right| \text{sign}(\vec{n} \cdot \nabla \eta)
\end{align*}
\]

where \( \psi = \begin{pmatrix} 1, u, v, (E + p) / \rho \end{pmatrix}^T \), \( P = (0, pn_x, pn_y, 0)^T \), with \( n_x \) and \( n_y \) specifying the face normal components in \( x \) and \( y \) directions. The superscript ‘i’ indicates the inviscid flux. The reconstruction of the viscous flux is based on a simple average of the ‘left’ and ‘right’ fluxes. The detailed reconstruction procedures are well stated in Ref. 23.

![Figure 2](image)

**Figure 2.** (a) Transformation from a moving physical domain to a fixed computational domain; (b) Distribution of solution points (circles) and flux points (squares) in a standard quadrilateral element for a third-order accurate SD scheme.

### C. Grid deformation strategy and simulation parameters

A snapshot of the deformation grid is displayed in Fig. 2. Herein, the pitching motion of the airfoil is controlled by a cosine algorithm as

\[
\alpha(t) = \alpha_m + \alpha_0 \cos(\omega t + \phi_0), \quad \omega = 2\pi f
\]
where $\alpha_m$ is the mean angle of attack, $\alpha_0$ is the amplitude of the pitching angle, and $\phi_0$ is the initial phase of the airfoil.

The rigid-body motion for the wall can be expressed as

$$\begin{align*}
    x_{\text{present}} - x_c &= (x_{\text{former}} - x_c) \cos(\Delta \alpha) - (y_{\text{former}} - y_c) \sin(\Delta \alpha) \\
y_{\text{present}} - y_c &= (x_{\text{former}} - x_c) \sin(\Delta \alpha) - (y_{\text{former}} - y_c) \cos(\Delta \alpha)
\end{align*}$$

(11)

where $(x_c, y_c)$ is the pitching center, and $\alpha = \alpha_0 \cos(\omega t + \phi_0)$.

Then by using a fifth-order polynomial blending function proposed in Ref. 35,

$$r_5(s) = 10s^3 - 15s^4 + 6s^5, s \in [0,1]$$

(12)

the motion of the boundary is interpolated to the whole inner grids. More details for the grid deformation strategy used in this study can be found in Ref. 24.

The chord length based Reynolds number ($R_e = \rho U_\infty C/\mu$) for the experiment is 3,340. The Strouhal number ($S_t$), which is the ratio of the characteristic velocity of the flapping wing to the free stream velocity, defined by $S_t = 2f A / U_\infty$, varies from 0.1 to 0.37 for the experimental study and some higher Strouhal numbers out of the aforementioned range are also tested in the numerical study. The reduced frequency ($K$), which is a measure of flapping frequency with respect to the intrinsic frequency of the flow over the airfoil, defined by $K = 2\pi f c / U_\infty$, varies from 5.0 to 17.6 in the experiments and some higher values are also used in the numerical simulations. Note that the parameter variation range is large for both the experimental and numerical studies. In the present study H-refinement (grid refinement) and p-refinement studies were conducted at first to determine the suitable grid and numerical accuracy. Based on the investigations, a 3rd order accurate scheme with a medium mesh was chosen as shown in Fig. 3. Details about these studies can be found in the Ref. 19.

**Figure 3.** Snapshot of the deformable grid. (a) is an overview of the grid and (b) is a zoom-in observation near the wall boundary.

**IV. Results and Discussions**

Both experimental and numerical results on wake structures behind a pitching NACA0012 airfoil will
be analyzed in this section. The comparison between numerical and experimental spanwise vorticity fields at \( S_c = 0.31, K = 15.1 \) and the pitching amplitude \( \alpha_0 = 5^\circ \) is shown in Fig. 4. It is clear that numerical results bare good visual agreements with the experimental results. Other validation of the numerical results against experimental data with different flow conditions and airfoil kinematics is available at Ref. 24.

A. Experimental results on the wake vortex structure transition process

In this section, two types of wake vortex structure transitions processes are emphasized at \( \alpha_0 = 5^\circ \), namely the transition from a drag-indicative wake to a thrust-indicative wake, and the transition from a symmetric wake to an asymmetric (deflective) wake. The asymmetric wake phenomenon in the present study has its special features are explained as follows. The oscillating airfoil’s shape and kinematics are symmetric with respect to a baseline. Here for the pitching motion, the baseline is the horizontal line passing through the axis of symmetry of the NACA0012 airfoil with a zero AoA. The phenomenon that the center line of the wake vortex street has a deflective angle with the baseline is named as the asymmetric wake phenomenon in this study. As discussed in the first section, this phenomenon is ‘abnormal’ as both the geometric and dynamic parameters of the airfoil are symmetric and for small Strouhal numbers, the wake vortex street is symmetric about the baseline. Explanations on this phenomenon will be shown later in the paper.

The wake transition processes from the drag-indicative type to the thrust-indicative type are displayed in Fig. 5-7. In each figure, vorticity fields behind the oscillating airfoil at four phases, namely \( 0^\circ, 90^\circ, 180^\circ \) and \( 270^\circ \), are shown. The corresponding velocity vectors for each phase are also plotted in these figures. Note that only a quarter of the total vectors are displayed here. From Fig. 5, it is clear that the vortex row with negative vorticity is on top of the vortex row with positive vorticity, resulting in a momentum deficit between the two rows. These vortex configurations form a drag-indicative wake. The Strouhal number and the reduced frequency for this case are 0.1 and 5.0 respectively. When the Strouhal number and the reduced frequency increase to 0.12 and 5.7, a ‘neutral’ wake appears. The feature of the wake is that the alternating sign vortices are almost aligned along a straight line as shown in Fig. 6. A thrust-indicative wake is generated when the Strouhal number and the reduced frequency further increase to 0.29 and 13.8. From Fig. 7, it is obvious that the vortex row with positive vorticity is on top of the vortex row with negative vorticity, resulting in a momentum surplus between the two rows. Note that all vortices in these three cases are equally spaced in the streamwise direction. The time-averaged velocity fields and spanwise vorticity fields for these three cases are shown in Fig. 8. The different features of the three wake types as aforementioned can be clearly concluded from the time-averaged velocity fields. It is observed that when the geometric and dynamic parameters of the airfoil are symmetric about the baseline, the drag-indicative wakes are also symmetric, but in the regime of the thrust-indicative wakes asymmetric wakes might occur.

The wake transition processes from the symmetric type to the asymmetric type are displayed in Fig. 9 and 10 with the spanwise vorticity fields. Note that the wake in Fig. 9 is slightly deflected from the baseline. The Strouhal number and the reduced frequency for this case are 0.31 and 15.1. From the figure it is also found that the distances between adjacent vortices with different sign vortices become a little different, and the two vortices shed in one pitching cycle have the tendency to move closer with each other. This transition point is smaller than the proposed threshold 0.434 of the deflected wake in Ref. 20. The reasons might be contributed to that the threshold proposed in Ref. 20 is for a plunging airfoil oscillating at a lower reduced frequency (\( \approx 6.3 \)). It becomes more apparent that when the Strouhal number and the reduced frequency reach up to 0.37 and 17.6, ‘dipole’ like vortex structures appear as shown in Fig. 10, and the dipole is made up of the two vortices shed in one pitching cycle. The corresponding time-averaged
velocity and spanwise vorticity fields are shown in Fig. 11. From the time-averaged velocity field it is clear that for the case with $S_t = 0.31$ and $K = 15.1$, the wake slightly goes up and for the case with $S_t = 0.37$ and $K = 17.6$ the wake has a large deflective angle with the baseline. Also note that thrust-indicative wakes can be observed from the time-averaged velocity and vorticity fields.

![Figure 4. Comparison between the phase-locked spanwise vorticity fields from PIV (a) and the instantaneous numerical vorticity fields (b) with $S_t = 0.31$, $K = 15.1$ and the pitching amplitude $\alpha_o = 5^\circ$ at the same phase.](image)

![Figure 5. Phase-locked spanwise vorticity fields from PIV with $S_t = 0.1$, $K = 5.0$ and the pitching amplitude $\alpha_o = 5^\circ$ at four different phases, namely $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$.](image)
Figure 6. Phase-locked spanwise vorticity fields from PIV with $S_h = 0.12$, $K = 5.7$ and the pitching amplitude $\alpha_0 = 5^\circ$ at four different phases, namely $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$.

Figure 7. Phase-locked spanwise vorticity fields from PIV with $S_h = 0.29$, $K = 13.8$ and the pitching amplitude $\alpha_0 = 5^\circ$ at four different phases, namely $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$. 
Figure 8. Time-averaged velocity (up) and vorticity (down) fields from the PIV measurements with the pitching amplitude $\alpha_0 = 5^\circ$. (a) $S_t = 0.1, K = 5.0$; (b) $S_t = 0.12, K = 5.7$; (c) $S_t = 0.29, K = 13.8$. 
Figure 9. Phase-locked spanwise vorticity fields from PIV with $S_t = 0.31, K = 15.1$ and the pitching amplitude $\alpha_0 = 5^\circ$ at four different phases, namely $0^\circ, 90^\circ, 180^\circ$ and $270^\circ$.

Figure 10. Phase-locked spanwise vorticity fields from PIV with $S_t = 0.37, K = 17.6$ and the pitching amplitude $\alpha_0 = 5^\circ$ at four different phases, namely $0^\circ, 90^\circ, 180^\circ$ and $270^\circ$.
Figure 11. Time-averaged velocity (up) and vorticity (down) fields from the PIV measurements with the pitching amplitude $\alpha_0 = 5^\circ$. (a) $S_t = 0.31$, $K = 15.1$; (b) $S_t = 0.37$, $K = 17.6$.

B. Experimental and numerical results on the deflective wake phenomenon

Before moving forwards, several questions might arise from the results about the asymmetric wake in the previous section. One burning question is: Can the deflective direction of the wake be changed? And if so, which factors will determine the deflective direction? As reviewed in the introduction part, different researchers found different features of the asymmetric wake. Some found that the deflective direction of the wake can change during one shoot of the experiment\textsuperscript{17} or the numerical simulation\textsuperscript{16}; some found that the deflective direction of the wake is random during the experiment\textsuperscript{14}; some confirmed that the wake deflective direction is determined by the initial conditions of the numerical simulations\textsuperscript{19,21}. In the present studies, different features of the asymmetric wakes have been found. The results are displayed as below.

In the experimental study, it seems that the wake deflective direction is determined by an unknown factor and the direction will keep the same in duplicated experiments. As a matter of fact, the wake direction sensitivity on the airfoil geometry, airfoil mounting method and linkage system and airfoil motion has been checked. However, it turns out that for the present study, all these factors have no effects on the wake deflective direction. Interestingly, the deflection of the asymmetric wake is found to be sensitive to the alignment of the wind tunnel and the wakes with different deflective directions were captured in two experiments. The vorticity fields for these two experiments at $S_t = 0.33$ and $K = 15.7$ are shown in Fig. 12. It is believed that the asymmetric wake is more sensitive to some unknown disturbances from the alignment of the wind tunnel in the present experiment than the considered parameters, and these unknown disturbances are amplified when the aerodynamic parameters, like the Strouhal number and the reduced frequency, exceed certain range (here $S_t = 0.31$ and $K = 15.1$). Finally the amplified disturbances induce
the deflective direction of the wake.

The wake deflective directions in the numerical simulations are determined by the initial pitching directions of the airfoil. The spanwise vorticity fields with the initial phases $\phi_0 = 0^\circ$ and $180^\circ$ are shown in Fig. 13. The Strouhal number and the reduced frequency for this case are 0.75 and 18.0 respectively. From the results shown in Fig. 13, it is found that the asymmetric wakes with different initial phases can have different deflected angles. Meanwhile, since the initial phases chosen in these two cases make the initial positions of the airfoil are symmetric to the horizontal axis of the airfoil, the wake shown at the same time is anti-symmetric. It can also be inferred that the thrust coefficient histories should be the same for the two cases, while the lift coefficient histories should be of the same absolute values but the opposite signs. This has been confirmed by the thrust and lift histories shown in Fig. 14. Note that from Fig. 13 and 14, when the wake is deflected upward, it will induce a low pressure region on the top surface of the airfoil, which could result in a positive lift. This has previously been reported by Cleaver et al.\textsuperscript{36}. Since leading edge vortices (LEVs) appear in this case, they will bring in disturbances into the flow fields. Whether these disturbances trigger the asymmetric wake or not needs to be examined. The results for the spanwise vorticity fields with the initial phases $\phi_0 = 0^\circ$ and $180^\circ$ at $St = 0.33$, $K = 23.0$ and $\alpha_0 = 2^\circ$ are shown in Fig. 15. It is clearly observed that even if there exists no LEVs, the asymmetric wake can occur. Therefore, it is quite possible that the asymmetric wake phenomenon is intrinsically an inviscid phenomenon and the vortex dynamics will dominate the formation of the vortex street structure although at high Strouhal numbers LEVs will introduce large disturbances to the wake structures and even affect the vortex structures much\textsuperscript{16}.

As discussed in Ref. 10, the threshold of the asymmetric wake could vary with the Strouhal number and the reduced frequency. Here three cases at the same Strouhal number (0.31) but different reduced frequencies, namely 15.1, 7.5 and 5.0, are numerically studied. The pitching amplitudes for these three cases are $5^\circ$, $10^\circ$ and $15^\circ$ respectively. From the spanwise vorticity fields and the corresponding time-averaged velocity fields as displayed in Fig. 16, it is found that the asymmetric wake is more apparent for the case with a larger reduced frequency. The deflective point for the case with $K = 15.1$ is around two-chord length downstream of the airfoil. This point is shifted to the position which is about three-chord length downstream of the airfoil for the case with $K = 7.5$. However, it is hard to distinguish the deflective point for the case with $K = 5.0$ even after five-chord length downstream of the airfoil. These phenomena can be explained like this. Theoretically, based on the definition of the Strouhal number, at the same Strouhal number, the vortex streets should have similar configurations --- the ratio between the distance of the two vortex rows and the distance of the adjacent two vortices with different sign vorticity is the same. However, if the reduced frequency is large, the strength of the shedding vortices will become large and the distance between the adjacent two vortices shed in one pitching cycle becomes small. This increases the chance for the interaction between these two vortices. As shown above, they have the tendency to form a dipole mode and then change the configuration of the whole vortex street. If the reduced frequency is small, the strength of the shedding vortices will become small and the distance between the adjacent two vortices shed in one pitching cycle becomes large. Both effects will reduce the chance for the formation of the asymmetric wake. The thrust and lift coefficient histories are displayed in Fig. 17. It is clear that at the present parameters, a relatively large reduced frequency is beneficial for the thrust generation while the wake structures almost have no effects on the lift production as no net lift has been generated.

V. Conclusions

PIV measurements have been performed for the wake structure analyses behind a sinusoidally pitching
NACA0012 airfoil. A high-order unstructured dynamic grid based SD compressible solver has been used to study the same wake structures. The experimental and numerical results bare good visual agreement with each other. Two types of wake transition processes, namely the transition from a drag-indicative wake to a thrust-indicative wake and that from the symmetric wake to the asymmetric wake are experimentally studied. Using the present geometric and dynamic parameters, the wake transition point from the drag-indicative type to the thrust-indicative type is found at $S_t = 0.12$; the transition point from the symmetric type to the asymmetric type is at $S_t = 0.31$. The asymmetric wake is believed to be closely related to the formation of a dipole-like vortex pair shed in one pitching cycle and can be treated as an inviscid phenomenon. In numerical simulations, the deflective angle of the asymmetric wake is determined by the initial phase angle, while in the experiments, some unknown disturbance determines the wake deflective direction. The reduced frequency will affect the strength of the shedding vortices and further affect the formation of the asymmetric wake.

References


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Figure 12. Phase-locked spanwise vorticity fields from PIV for two asymmetric wakes with different deflective directions at $S_t = 0.37, K = 17.6$ and the pitching amplitude $\alpha_0 = 5^\circ$.

Figure 13. Spanwise vorticity fields from numerical simulations with $S_t = 0.75, K = 18.0$ and the pitching amplitude $\alpha_0 = 10^\circ$ at the same time. The initial phase angles are (a) $\phi_0 = 180^\circ$ and (b) $\phi_0 = 0^\circ$. 
Figure 14. Aerodynamic force coefficient histories for the pitching airfoil with $S_t = 0.75$, $K = 18.0$ and the pitching amplitude $\alpha_0 = 10^\circ$ and different initial phase angles $0^\circ$ and $180^\circ$. (a) Thrust coefficient histories; (b) lift coefficient histories.

Figure 15. Spanwise vorticity fields from numerical simulations with $S_t = 0.33$, $K = 23.0$ and the pitching amplitude $\alpha_0 = 2^\circ$ at the same time. The initial phase angles are (a) $\phi_0 = 180^\circ$ and (b) $\phi_0 = 0^\circ$. 
Figure 16. Instantaneous spanwise vorticity fields and time-averaged velocity fields with $S_t = 0.31$. (a) $K = 15.1$ and the pitching amplitude $\alpha_0 = 5^\circ$; (b) $K = 7.5$ and the pitching amplitude $\alpha_0 = 10^\circ$; (c) $K = 5$ and the pitching amplitude $\alpha_0 = 15^\circ$. The phase angle for the three vorticity fields is $0^\circ$.

Figure 17. Aerodynamic force coefficient histories for the pitching airfoil with $S_t = 0.31$ but different reduced frequencies and pitching amplitudes. (a) Thrust coefficient histories; (b) lift coefficient histories.