Measuring MPI Latency and Communication Rates for Small Messages
by
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Abstract. The authors present a method for measuring MPI latency and communication rates for small messages. They first investigate the overhead and resolution of the wall-clock timer and then applying this to measure ping pong times with small numbers of ping pongs within each timing loop instead of using the traditional thousands of ping pongs within a timing loop. Since timings vary, standard statistical methods are used to analyze the timing data and the minimum, median, average and maximum timings are calculated as well as the standard deviation, standard error and relative standard error so one can understand the distribution of the timing data. The program that performs these calculations is freely available and can be downloaded by going to http://hpcgroup.public.iastate.edu/.

1. Introduction

Currently, most scientific programs written for distributed memory parallel computers use the MPI message-passing library [1]. The speed that messages can be sent between processors using MPI routines is an important performance criterion for a parallel computer. Message passing performance between two nodes or within a node of a computer is commonly determined by measuring ping pong times when sending messages of various sizes. The operation of sending a message from one MPI process to another and back using blocking MPI sends and receives (i.e. mpi_send and mpi_recv) is usually called the ping pong operation. MPI latency is usually defined to be half of the time of a ping pong operation with a message of size zero.

The time to execute the ping pong operation for small messages on many of today’s high performance communication networks is so small that the effects of the various software layers (e.g. OS, network card drivers, the software on the communication network) may cause ping pong times to vary significantly. Because of this, one should use standard statistical methods to describe the distribution of the timings such as minimum, mean, median, maximum, standard deviation, standard error and relative standard error. It is important to know the distribution of ping pong times and not just the average since large variations in ping pong times may result in poor load balancing and poor scalability for applications which spend significant amounts of time sending and receiving small messages.

Ideally, one would like an infinitely precise wall-clock timer to measure each ping pong time to obtain the distribution of ping pong times. However, on many of today’s computers wall clock timers (e.g. mpi_wtime) are not able to accurately measure small ping pong times. For example, for the Opteron/InfiniPath cluster, named lightning, at Iowa State University (ISU), the resolution of mpi_wtime is about 1 microsecond and the average MPI ping pong latency times are around 1.3 microseconds (about 2.6 microseconds for a round trip). This lack of timer precision is usually
overcome by timing many (often thousands of) ping pongs within a single timing loop and then taking the average. However, taking an average of thousands of ping pongs within a single timing loop hides the distribution of the individual ping pong times. To better understand the distribution of ping pong times, the authors propose to not measure times with thousands of ping pongs in a single timing loop but to greatly reduce the number of ping pongs in a timing loop to a value based on the resolution of the timer and on a median ping pong time.

Communication rates between nodes depend on the size of the message being sent and are an important performance criterion of a parallel computer. For this study, uni-directional communication rates are measured using the ping pong operation with blocking MPI sends and receives. One can measure bi-directional communication rates by using either mpi_sendrecv or nonblocking MPI sends and receives [2]. Currently, the benchmarks in [2] do not perform multiple timing trials for statistical analysis.

A program has been written that automatically measures and statistically analyzes timing data for the ping pong operation using MPI blocking sends and receives. The program that performs these calculations is freely available and can be downloaded by going to http://hpcgroup.public.iastate.edu/. Requests to have results posted on this website should be sent to Marina Kraeva at kraeva@iastate.edu.

2. Related Work

Currently, MPI latency and communication rates are usually measured using either the Ohio State University’s OMB benchmarks [2] or Intel’s MPI Benchmark [4] (formerly the PALLAS MPI test suite). The OMB benchmarks take an average of 10,000 ping pongs and the Intel benchmarks take an average of 1,000 ping pongs. Neither measures multiple timings for a single message size and no statistical analysis of timings is performed. We found it to be inconvenient that the OMB benchmark program is distributed with ping pongs between processors of rank 0 and 1. Intel’s MPI Benchmark measures ping pong times between processors of rank 0 and p-1, where p is the number of MPI processes used when running the program. For the machines that the authors used, ping pongs between processors of rank 0 and p-1 were convenient because of the way MPI processes were distributed among nodes. For convenience, the program written for this paper measures ping pong times between MPI processes of rank “source” and “dest” where these values are set at the beginning of the program.

The seventh benchmark in the HPC Challenge Benchmark [3] is designed to measure the time required to perform simultaneous communication patterns and is based on the effective bandwidth benchmark, $b_{eff}$. The effective bandwidth benchmark measures the accumulated bandwidth of the communication network of parallel and/or distributed computing systems. Several message sizes, communication patterns and methods are used. The algorithm uses an average to take into account that short and long messages are transferred with different bandwidth values in real applications. These tests are not designed to measure latency and communication rates of small messages between two MPI processes.
3. Methodology

3.1 Measuring Timer Overhead and Resolution
Since timings are done with the MPI wall-clock timer, mpi_wtime, we first investigate the resolution and overhead of mpi_wtime. When timing ping pong operations, timings are performed on the processor from which the ping pong operation begins. The elapsed time of ping-pongs is measured in the standard way, i.e.

\[
\begin{align*}
t_1 &= \text{mpi\_wtime}() \\
\ldots &\text{ping-pongs} \ldots \\
t_2 &= \text{mpi\_wtime}() \\
\text{time} &= t_2 - t_1
\end{align*}
\]

To investigate the resolution and overhead of mpi_wtime, we measure the times when there are no statements between timer calls, i.e.

\[
\begin{align*}
do\ i &= 1, n \\
t_1 &= \text{mpi\_wtime}() \\
t_2 &= \text{mpi\_wtime}() \\
\text{time}(i) &= t_2 - t_1 \\
\end{align*}
\]

Experiments run on different machines showed a large variation in measured times so experiments were performed with a large number of timing trials to investigate the distribution of timings. With 2**24 timing trials, the following timing distribution was obtained using ISU’s lightning cluster running RedHat Enterprise Linux 4.4 (Linux kernel 2.6.9-42.ELsmp). Times are in microseconds.

\[
\begin{align*}
\text{minimum time} &= 0.95 \\
\text{median time} &= 1.91 \\
\text{average time} &= 1.62 \\
\text{maximum time} &= 12500.05 \\
\text{standard deviation} &= 1.61
\end{align*}
\]

Maximum times varied significantly from one run to another. For example, when making 10 runs with n = 2**24 the following maximum times were obtained: 161, 162, 266, 280, 327, 363, 723, 743, 891 and 12500 microseconds. The large variation in maximum times is likely due to interference from the operating system.

Average times over these same 10 runs varied from 1.62 to 1.75 microseconds. The minimum and median times were the same for all runs made (over 60 runs were made) when times were rounded as shown above. Appendix A shows the results of one of these runs and shows the complete distribution of all measured times.

When timing events of short duration, one should take the above timing overhead and timing variation into consideration. We therefore define the resolution of the above timer, res_timing, to be the minimum positive time of the times time(i), i = 1, n. Using the Fortran minval intrinsic function, this becomes:

\[
\text{res\_timing} = \text{minval(time(1:n), mask = time(1:n) > 0.d0)}
\]
Experiments show that taking \( n = 2^{24} \) was large enough to capture the minimum positive time. It is unlikely with this large value for \( n \) that all the measured times are zero, but if this does happen we define \( \text{res\_timing} \) to be zero.

Define the **minimum timing overhead, \( \text{min\_ov} \),** to be the minimum of the non-negative times:

\[
\text{min\_ov} = \minval(\text{time}(1:n), \text{mask} = \text{time}(1:n) \geq 0.d0)
\]

The minimum timing overhead is then subtracted from the elapsed time of “code-to-be-timed”:

\[
\begin{align*}
\text{t1} &= \text{mpi\_wtime()} \\
\ldots & \text{code-to-be-timed} \ldots \\
\text{t2} &= \text{mpi\_wtime()} \\
\text{time} &= \text{t2} - \text{t1} - \text{min\_ov}
\end{align*}
\]

Notice that subtracting the average or median time instead of the minimum time might lead to some times being negative.

### 3.2 Timing Methodology

When measuring ping pong times the minimum timing overhead is subtracted from the measured times. This is applied to measuring “npp” ping pongs within a single timing loop as follows:

```plaintext
do ktrial = 1, ntrial
    call mpi\_barrier(comm, ierr)
    if (rank == source) then
        t1 = mpi\_wtime()
        do i = 1, npp
            call mpi\_send(A(1), n, dp, dest, 1, comm, ierr)
            call mpi\_recv(A(1), n, dp, dest, 1, comm, mpi\_status\_ignore, ierr)
        enddo
        t2 = mpi\_wtime()
        time(ktrial) = t2 - t1 - min\_ov
    endif
    if (rank == dest) then
        do i = 1, npp
            call mpi\_recv(A(1), n, dp, source, 1, comm, mpi\_status\_ignore, ierr)
            call mpi\_send(A(1), n, dp, source, 1, comm, ierr)
        enddo
    endif
enddo
```

where “\( dp \)” is “mpi\_double\_precision” and the ping pongs are between MPI processes of rank “source” and “dest”.

Notice that the source process may release from mpi\_barrier before the dest process. This can cause unnecessary variability in the measured timing data. To lessen this variability, a hand-shaking procedure was added by calling mpi\_recv before the call to the timer on the rank = source process and calling mpi\_send on the rank = dest process as follows: (The two additional hand-shaking statements are highlighted in bold.)
do ktrial = 1, ntrial
   call mpi_barrier(comm, ierror)
   if (rank == source) then
      call mpi_recv(A(1), 1, dp, dest, 1, comm, mpi_status_ignore, ierror)
      t1 = mpi_wtime()
      do i = 1, npp
         call mpi_send(A(1), n, dp, dest, 1, comm, ierror)
         call mpi_recv(A(1), n, dp, dest, 1, comm, mpi_status_ignore, ierror)
      enddo
      t2 = mpi_wtime()
      time(ktrial) = t2 - t1 - min_ov
   endif
   if (rank == dest) then
      call mpi_send(A(1), 1, dp, source, 1, comm, ierror)
      do i = 1, npp
         call mpi_recv(A(1), n, dp, source, 1, comm, mpi_status_ignore, ierror)
         call mpi_send(A(1), n, dp, source, 1, comm, ierror)
      enddo
   endif
enddo

Notice that this hand-shaking does not guarantee that the MPI receive is posted before the ping pong
operation begins. For example, it could happen that a long interruption occurs right after the mpi_send
is executed on the processor of rank = dest. However, experiments show that using this hand-shaking
does lower the variability of the timing data.

3.3 Estimating npp - the number of ping pongs in a timing loop
Let npp be the number of ping pongs within each timing loop. Ideally one would like to precisely
measure the time of a single ping pong many times and then statistically analyze the timing data.
However, this is not possible due to the lack of timer precision, especially when ping pong times are of
the same order as the resolution of the timer. Thus, we have the following dilemma:
- If npp is large, then we can accurately measure timing trials, but the information about the
  variability of each timing trial is unknown.
- If npp is small, then the information about the variability of the timing trials is known, but these
  times cannot be measured accurately.

We now present a method for choosing npp in a way that takes into account both the resolution of the
timer and a median time for the ping pong. To illustrate this method, suppose the resolution of the
timer is 1 microsecond and that the time for a single, round trip ping pong time is 2 microseconds. For
this case, we suggest choosing npp to be 25 so that there will be about 50 = 50*(res_timing)
microseconds in each timing loop. There was much discussion among the authors regarding whether
we should recommend a factor of 10, 25, 50, 100 or something else. We also discussed choosing npp
so the standard deviation for the timing trials was less than some value. In the end, the authors decided
to recommend that npp be large enough so that timings were “accurate” within 50*(res_timing). In
the program this factor is named res_npp and it can be easily set to any positive integer.

The MPI API says that the MPI function, mpi_wtick, is to return the value of the clock tick for the MPI
wall-clock timer, mpi_wtime. Initially, we considered using the value of the clock tick to approximate
npp. However, we could not use mpi_wtick since we discovered that the value returned by mpi_wtick was not constant on some machines.

The estimated value for npp is calculated as follows. Let npp_init be an initial value given npp to calculate an approximation for a single ping pong time. Let the double precision array npptime(1:npptrial) be the measured times for “npptrial” timings with npp_init ping pongs within a single timing loop. We now need to decide whether to use the minimum, median or average of these times to calculate an approximation for the ping pong time. Experiments show that the median time is the most stable when making multiple timing trials, so this is what is used. Then the median time for one ping pong, median_ppt is

\[
\text{median}\_\text{ppt} = \text{median}(\text{npptime}(1:\text{npptrial})/\text{dble(npp\_init)})
\]

Using res_timing defined above, one would like

\[
npp*\text{median}\_\text{ppt} \geq (\text{res\_npp})*(\text{res\_timing})
\]

Dividing the above inequality by median_ppt gives

\[
npp \geq (\text{res\_npp})*(\text{res\_timing})/\text{median}\_\text{ppt}
\]

It might happen that the right hand side of the above inequality is less than 1.d0. When this happens, npp should have the value of 1. Thus,

\[
npp = \max(1.d0, (\text{res\_npp})*(\text{res\_timing})/\text{median}\_\text{ppt})
\]

Since the right hand side of the above equation may not have an integer value and since npp must be a positive integer, we use the nearest integer intrinsic function, nint, and define

\[
npp = \text{nint}(\max(1.d0, (\text{res\_npp})*(\text{res\_timing})/\text{median}\_\text{ppt}))
\]

Occasionally the computed value of npp may change from one run to another due to small variations in the values of res_timing and/or median_ppt. To be able to compare timing data from one run to another, one should do this with the same value of npp. Thus (as has already been mentioned), this program has the option of setting the value of npp to be any positive integer instead of having its value calculated. This option also allows one to explore the effects of trying different values of npp.

### 3.4 Analysis of Ping Pong Timing Data

After the ping pong timing data has been collected, it is then analyzed using standard statistical techniques [5]. Specifically, in addition to calculating the minimum, median, average and maximum times, we also calculate the variance, standard deviation, coefficient of variation, standard error, relative standard error (i.e. the standard error divided by the average) and the distribution of the timing data. Communication rates using the minimum, median and average times are also calculated. Appendix B shows the distribution of ping pong times from the lightning Opteron/InfiniPath cluster at Iowa State University when npp = 1. Appendix C gives the summary output from a run on the lightning Opteron/InfiniPath cluster at Iowa State University with npp = 18.

When there are a large number of times much greater than the median time, then these will increase the average value. Statistically these large values could be considered as “outliers” that should be filtered out. The program written by the authors not only statistically analyzes the unfiltered timing data but
does this same analysis on the filtered timing data when removing all timing trials that are larger than 2 times the median. This cut off value is set via the “cut_coef” and can be easily changed. Filtering the timing data stabilizes the average value, but does not allow one to know the variability of the timing data.

4. Results

4.1 Lightning timing results using the InfiniPath/HTX interconnect
Recall that ntrial is the number of timing trials and that npp is the number of ping pongs in a timing loop. Table 4.1 compares the timing results for a message of size 8 bytes when running on the lightning cluster with the InfiniPath/HTC interconnect with ntrial at least 64 million when npp = 1, 9, 18, 36, 72 and 144. When npp = 15000, ntrial was 15000. The values of npp = 9, 18, 36, 72 and 144 correspond to the calculated value of npp when res_npp = 25, 50 (the recommended value), 100, 200 and 400, respectively. npp = 15000 was chosen since standard latency benchmarks time 10 to 15 thousand ping pongs and measure a single time instead of making multiple timings with a timing loop.

<table>
<thead>
<tr>
<th>npp</th>
<th>1</th>
<th>9</th>
<th>18</th>
<th>36</th>
<th>72</th>
<th>144</th>
<th>15000</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>1.43</td>
<td>1.22</td>
<td>1.25</td>
<td>1.26</td>
<td>1.36</td>
<td>1.36</td>
<td>1.37</td>
</tr>
<tr>
<td>med</td>
<td>2.03</td>
<td>1.34</td>
<td>1.31</td>
<td>1.31</td>
<td>1.40</td>
<td>1.38</td>
<td>1.38</td>
</tr>
<tr>
<td>avg</td>
<td>1.81</td>
<td>1.36</td>
<td>1.32</td>
<td>1.32</td>
<td>1.40</td>
<td>1.39</td>
<td>1.38</td>
</tr>
<tr>
<td>max</td>
<td>306.96</td>
<td>1133.88</td>
<td>570.81</td>
<td>284.50</td>
<td>45.79</td>
<td>26.06</td>
<td>2.43</td>
</tr>
<tr>
<td>Var</td>
<td>0.21</td>
<td>0.27</td>
<td>0.15</td>
<td>0.06</td>
<td>0.0037</td>
<td>0.0022</td>
<td>0.00010</td>
</tr>
<tr>
<td>SD</td>
<td>0.46</td>
<td>0.52</td>
<td>0.38</td>
<td>0.24</td>
<td>0.060</td>
<td>0.046</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Table 4.1. Lightning using the HTX interconnect: ping pong times in microseconds divided by 2 for 8 byte messages.

Notice that as npp increases, the variability of the measured times decreases and this hides the variability of the ping pong times. The variability of the timing data is important to know since this may affect program load balancing and scalability. Notice that even when npp = 15,000 the measured times vary from 1.37 to 2.43 microseconds. For the lightning cluster, the resolution of the timer is about 1 microsecond and the median (round trip) ping pong time is about 2.6 microseconds, so the accuracy of the median and average times is questionable when npp = 1.

Appendix B shows the distribution of measured times divided by 2 for the npp = 1. Notice that one can see the granularity of the timer, mpi_wtime. Appendix C shows the summary output file produced when npp = 18.

4.2 Lightning timing results using the Gigabit Ethernet interconnect
Table 4.2 compares the timing results for a message of size 8 bytes when running on the lightning cluster using the gigabit Ethernet interconnect with ntrial large enough so the execution time of the program takes about one hour. For example, when npp=1, ntrial=2**20 and when npp=15000, ntrial=2**10. The calculated value of npp was 1 with res_npp = 50. The gigabit Ethernet switch consists of four 48-port stackable switches. Tests were run on compute nodes connected to adjacent ports on a single 48-port switch.
Table 4.2. Lightning using the Gigabit Ethernet interconnect: ping pong times in microseconds divided by 2 for 8 byte messages.

As was the case in Table 4.1, as npp increases the variability of the measured times decreases hiding the variability of the ping pong times. Notice that even when npp = 15000 the measured times vary from 54.6 to 60.6 microseconds.

4.3 SSC’s Magic Cube timing results using an InfiniBand interconnect

The Magic Cube computer at the Shanghai Supercomputing Center (SSC) consists of 1500 nodes with each node having four AMD quad core, 1.9GHz Opteron (8347) processors sharing 64 GB of memory. Nodes are interconnected with an InfiniBand switch. The InfiniBand (IB) switch has two levels with every 10 compute nodes connected to a 24 port IB switch and the 24 port IB switches connected to upper level IB switches. Timings were measured between 2 nodes on a single 24 port IB switch. Magic Cube is running SuSe SLSE 10 SP2 and the compiler used is gcc version 4.1.2 20070115 (SUSE Linux) and gfortran 4.1.2 (SUSE Linux). MPI used is MVAPICH 1.1 compiled with gcc.

Table 4.3 compares the timing results for a message of size 8 bytes when running on Magic Cube using the InfiniBand interconnect with ntrial large enough so the execution time of the program takes about one hour. For example, when npp=1, ntrial =2**20 and when npp=15000, ntrial=2**10. The calculated the value of npp was 16 with res_npp = 50.

Table 4.3. Magic Cube with InfiniBand: ping pong times in microseconds divided by 2 for 8 byte messages.

As was the case in Table 4.1, as npp increases the variance and standard deviation of the measured times decreases hiding the variability of the ping pong times.
5. Conclusions

The authors present a method for measuring MPI latency and communication rates for small messages. When investigating the overhead and resolution of the timer, they found large variations in measured times when there were no statements between consecutive timer calls. Using this timing information, the authors present a method for timing ping pong operations that uses a small number of ping pongs within a timing loop instead of the traditional methods that use thousands of ping pongs in a timing loop. The small number of ping pongs in a timing loop is determined by the resolution of the timer and by an approximation to the time of a single ping pong. Having a small number of ping pongs in a timing loop allows one to better understand the variability of ping pong times. The distribution of timing data is calculated and then analyzed using standard statistical methods.

A program has been written that automatically measures and statistically analyzes timing data for the ping pong operation. The program allows one to select any value for the number of timing trials and the number of ping pongs in the timing loop. When the program is run, two files are produced: latency.output and latency.summary.output. The first file contains all the output produced and the second is a summary of the results. The program that performs these calculations is freely available and can be downloaded by going to http://hpcgroup.public.iastate.edu/. Requests to have results posted on this website should be sent to Marina Kraeva at kraeva@iastate.edu.

References
2. The Ohio State University Benchmarks: http://nowlab.cse.ohio-state.edu/projects/mpi-iba/benchmarks.html

Appendix A: Distribution of timing in time(1:n) on the ISU’s lightning Opteron/InfiniPath cluster when running the following with n = 2**24

do i = 1, n
   t1 = mpi_wtime()
   t2 = mpi_wtime()
   time(i) = t2 – t1
endo

DISTRIBUTION OF TIMES FROM THE TIMER OVERHEAD MEASUREMENTS IN PART 1:

<table>
<thead>
<tr>
<th>Time Range</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 &lt;= times &lt; 0.20</td>
<td>0</td>
</tr>
<tr>
<td>0.20 &lt;= times &lt; 0.40</td>
<td>0</td>
</tr>
<tr>
<td>0.40 &lt;= times &lt; 0.60</td>
<td>0</td>
</tr>
<tr>
<td>0.60 &lt;= times &lt; 0.80</td>
<td>0</td>
</tr>
<tr>
<td>0.80 &lt;= times &lt; 1.00</td>
<td>5118946</td>
</tr>
<tr>
<td>1.00 &lt;= times &lt; 1.20</td>
<td>1234766</td>
</tr>
<tr>
<td>1.20 &lt;= times &lt; 1.40</td>
<td>0</td>
</tr>
<tr>
<td>1.40 &lt;= times &lt; 1.60</td>
<td>0</td>
</tr>
<tr>
<td>1.60 &lt;= times &lt; 1.80</td>
<td>0</td>
</tr>
</tbody>
</table>
1.80 <= times < 2.00 with count = 6351041
2.00 <= times < 2.20 with count = 4035896
2.20 <= times < 2.40 with count = 0
2.40 <= times < 2.60 with count = 0
2.60 <= times < 2.80 with count = 0
2.80 <= times < 3.00 with count = 3665
3.00 <= times < 3.20 with count = 5052
3.20 <= times < 3.40 with count = 0
3.40 <= times < 3.60 with count = 0
3.60 <= times < 3.80 with count = 0
3.80 <= times < 4.00 with count = 6
4.00 <= times < 4.20 with count = 37
4.20 <= times < 4.40 with count = 0
4.40 <= times < 4.60 with count = 0
4.60 <= times < 4.80 with count = 1
4.80 <= times < 5.00 with count = 0
5.00 <= times < 5.20 with count = 13
5.20 <= times < 5.40 with count = 0
5.40 <= times < 5.60 with count = 0
5.60 <= times < 5.80 with count = 0
5.80 <= times < 6.00 with count = 222
6.00 <= times < 6.20 with count = 46
6.20 <= times < 6.40 with count = 0
6.40 <= times < 6.60 with count = 0
6.60 <= times < 6.80 with count = 0
6.80 <= times < 7.00 with count = 997
7.00 <= times < 7.20 with count = 522
7.20 <= times < 7.40 with count = 0
7.40 <= times < 7.60 with count = 0
7.60 <= times < 7.80 with count = 0
7.80 <= times < 8.00 with count = 1291
8.00 <= times < 8.20 with count = 1600
8.20 <= times < 8.40 with count = 0
8.40 <= times < 8.60 with count = 0
8.60 <= times < 8.80 with count = 0
8.80 <= times < 9.00 with count = 242
9.00 <= times < 9.20 with count = 732
9.20 <= times < 9.40 with count = 0
9.40 <= times < 9.60 with count = 0
9.60 <= times < 9.80 with count = 10
9.80 <= times with count = 22131

FREQUENCY OF TIMES FROM THE TIMER OVERHEAD MEASUREMENTS IN PART 1:
time = 0.954 with frequency = 5118946
time = 1.192 with frequency = 1234766
time = 1.907 with frequency = 6351041
time = 2.146 with frequency = 4035896
time = 2.861 with frequency = 3665
time = 3.099 with frequency = 5052
time = 3.815 with frequency = 6
time = 4.053 with frequency = 37
time = 4.768 with frequency = 1
time = 5.007 with frequency = 13
time = 5.960 with frequency = 222
time = 6.199 with frequency = 46
<table>
<thead>
<tr>
<th>Time</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.914</td>
<td>997</td>
</tr>
<tr>
<td>7.153</td>
<td>522</td>
</tr>
<tr>
<td>7.868</td>
<td>1291</td>
</tr>
<tr>
<td>8.106</td>
<td>1600</td>
</tr>
<tr>
<td>8.821</td>
<td>242</td>
</tr>
<tr>
<td>9.060</td>
<td>732</td>
</tr>
<tr>
<td>9.775</td>
<td>10</td>
</tr>
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Appendix B: The following distribution of ping pong timing data is from the lightning Opteron/InfiniPath cluster at Iowa State University and is the distribution of measured times divided by 2 for the npp=1 case. Times are in microseconds and the number of timing trials = 2**25.

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Appendix C: The summary output from a run on the lightning Opteron/InfiniPath cluster at Iowa State University with npp = 18 and the number of timing trials = 2**23.

PROGRAM PARAMETERS
Number of MPI processes used = 8
Ping pongs are between processors of rank source and dest where
  source = 0
  dest = 7
Number of timing trials used for parts 1, 2 & 3 are:

\[ \begin{align*} 
\text{ntr} & = 16777216 \\
\text{npptrial} & = 8388608 \\
\text{ntrial} & = 8388608 
\end{align*} \]

Message size(# of 8-bytes els) = 1

Number of ping pongs in a timing loop (npp):

- calculated value of npp = 18
- value of npp used = 18

TIMING RESULTS FOR PING PONG TIMES DIVIDED BY TWO IN MICROSECONDS

\[ \begin{align*} 
\text{minimum time} & = 1.2451 \\
\text{median time} & = 1.3113 \\
\text{average time} & = 1.3227 \\
\text{maximum time} & = 570.8072 
\end{align*} \]

STATISTICS FOR PING PONG TIMES

\[ \begin{align*} 
\text{variance} & = 0.1463E+00 \\
\text{standard deviation} & = 0.3825E+00 \\
\text{coefficient of variation} & = 0.2892E+02 \\
\text{standard error} & = 0.1321E-03 \\
\text{relative standard error} & = 0.9985E-04 
\end{align*} \]

COMMUNICATION RATE RESULTS IN MBYTES/SECOND

- communication rate using minimum time = 0.6425E+01
- communication rate using median time = 0.6101E+01
- communication rate using average time = 0.6048E+01
- communication rate using maximum time = 0.1402E-01

FILTERED DATA RESULTS FROM PART 5

- filtering cut-off coefficient = 2.0000
- timing trials after filtering = 8387483
- # of timings removed by filtering = 1125

FILTERED TIMING RESULTS FOR PING PONG TIMES DIVIDED BY TWO IN MICROSECONDS

\[ \begin{align*} 
\text{minimum time} & = 1.2451 \\
\text{median time} & = 1.3113 \\
\text{average time} & = 1.3219 \\
\text{maximum time} & = 2.6160 
\end{align*} \]

STATISTICS FOR FILTERED TIMINGS

\[ \begin{align*} 
\text{variance} & = 0.1207E-02 \\
\text{standard deviation} & = 0.3473E-01 \\
\text{coefficient of variation} & = 0.2628E+01 \\
\text{standard error} & = 0.1199E-04 \\
\text{relative standard error} & = 0.9073E-05 
\end{align*} \]

- program time in minutes = 0.4683E+02