1 Concurrent Users

A network provider investigates the load of its network. The number of concurrent users is recorded at fifty locations (thousands of people),

17.2 22.1 18.5 17.2 18.6 14.8 21.7 15.8 16.3 22.8
24.1 13.3 16.2 17.5 19.0 23.9 14.8 22.2 21.7 20.7
13.5 15.8 13.1 16.1 21.9 23.9 19.3 12.0 19.9 19.4
15.4 16.7 19.5 16.2 16.9 17.1 20.2 13.4 19.8 17.7
19.7 18.7 17.6 15.9 15.2 17.1 15.0 18.8 21.6 11.9

(a) Compute the sample mean, variance (unbiased), and standard deviation of the number of concurrent users.

(b) Compute the interquartile range. Are there any outliers?

(c) Compute the five-point summary and construct a boxplot.

(d) It is reported that the number of concurrent users follows approximately a Normal distribution. Does the histogram support this claim?

(8 points)

2 Parameter Estimation

Estimate the unknown parameter $\theta$ from a sample

3, 3, 3, 3, 7, 7, 7

drawn from a discrete distribution with the probability mass

$P(3) = \theta$

$P(7) = 1 - \theta$

Obtain two estimators for $\theta$:

(a) the method of moments estimator,

(b) the maximum likelihood estimator.

(4 points)

3 Parameter Estimation - Again

Use method of moments and method of maximum likelihood to estimate

(a) parameters $a$ and $b$ from a sample of size $n$ of a Uniform distribution $U(a, b)$. 

(b) parameter $\lambda$ from a sample of size $n$ of an Exponential distribution $\text{Exp}_\lambda$.

(c) parameter $\mu$ from sample of size $n$ of a Normal distribution $N_{\mu,\sigma^2}$, where we know $\sigma$.

(d) parameter $\sigma^2$ from a sample of size $n$ of a Normal distribution $N_{\mu,\sigma^2}$, where we know $\mu$.

(e) parameters $\mu, \sigma^2$ from a sample of size $n$ of a Normal distribution $N_{\mu,\sigma^2}$, where both $\mu$ and $\sigma^2$ are unknown.

(7.5 points)

4 Normal Distribution

Installation of a certain hardware takes random time with a standard deviation of 5 minutes.

(a) A computer technician installs this hardware on 64 different computers with an average installation time of 42 minutes. Give a 95% confidence interval for the mean installation time.

(b) Suppose that the population mean installation time is 40 minutes (true value of $\mu$). A technician installs the hardware on your PC. What is the probability that the installation time will be within the interval computed in (a)?

(4 points)

5 Normal Distribution - Again

Assume, salaries of entry level computer engineers have a Normal distribution with unknown mean and standard deviation. Three randomly selected computer engineers have salaries (in $1000s):

30, 50, 70

(a) Construct a 90% confidence intervals for the average salary of an entry level computer engineer.

(b) Does this sample provide significant evidence (at a 90% confidence level) that the average salary of entry level computer engineers is different from $80.000? Explain.

(4 points)