1 Playing Poker

The game of Poker uses a standard deck of 52 cards. For each hand, 5 cards are drawn at random.

(a) Sketch out a sample space for drawing five cards out of 52 cards at random. Determine its size.

(b) A flush is a set of five cards of the same suit (a suit is one of the following groups: hearts, spades, diamonds, or clubs). How many different ways are there to randomly draw a flush? What is its probability?

(c) What is the probability of a hand of four of a kind (i.e. four fives, four sixes, four sevens, ...)?

2 Rolling Dice

Many games involve rolling dice.

(a) Sketch the sample space for rolling $k$ dice (assume $k$ is a smallish integer). What is the size of this sample space, if you assume that order does not matter? What, if you assume that order does matter?

(b) Assume, only the number of 6s that have been rolled with $k$ dice is important. What is the probability that at least one 6 is rolled with $k$ dice?

(c) Now assume that you order the $k$ dice according to outcome. What is the probability that 6 is the highest number? What is the probability that 5 is the highest number? For each possible outcome, compute its probability.

3 Languages

Among employees of a certain firm, 70% know C/C++, 60% know Fortran, and 50% know both. What proportion of programmers

(a) does not know Fortran?

(b) does not know Fortran and C/C++?

(c) knows Fortran but not C/C++?

(d) knows C/C++, if they know Fortran?

(e) knows Fortran, if they know C/C++?
4 Practising Counting

(a) A soccer team has 11 players. Before a game starts, the players come out of the team box one after the other. How many different orders are possible?

(b) The coach has to decide on five players when the game comes to a penalty shoot-out. How many possibilities does he have?

(c) The coach has to decide on five players when the game comes to a penalty shoot-out and give an order, in which they have to shoot. How many possibilities does he have?

(d) Ten players form two basketball teams. How many possibilities for building different teams are there?

(e) Some student club consists of 5 statisticians, 10 computer scientists and 15 mathematicians. For a committee six students are elected, two from each department. How many different possibilities for the committee members do exist?

(f) Egg Hunt: How many different possibilities are there to distribute 20 different eggs to three nests?

(g) On a party six people shake hands: each one shakes hands with everybody else (only once). How many handshakes does that make in total?

(h) A delivery of 50 transistors contains 40 good ones and 10 defectives. In a test five of them are checked. How many possibilities are there to have 3 good ones and 2 defective transistors in the test set?

(i) Someone has 15 books - 3 on cooking, 5 on music and 7 novels. How many ways does Someone have to arrange the books on a shelf, if books on the same topic are supposed to be together?

(j) How many ways are there to arrange the letters CHOCOLATECHIPCOOKIES to different "words"?

5 System Reliability

Calculate the reliability of each of the systems shown in the figure, if components $A, B, C, D,$ and $E$ function properly (and independently of each other) with probabilities 0.9, 0.8, 0.7, 0.6, and 0.5, respectively.

(5 points)