Quiz 8 Solution - Math 166

Name: ____

Show and justify **all** work to receive maximum credit for each problem. You may not use your book, notes, or a calculator on this quiz. Give exact answers, not decimal approximations. Do not give answers as mixed fractions. This quiz is worth 20 points.

If you use a test for convergence or divergence of a series, state which test you are using and why it applies.

1. Use the Ordinary Comparison Test to determine if the series $\sum_{n=1}^{\infty} \frac{n^2}{3n^3-2}$ converges or diverges.

   Solution: Since $\frac{n^2}{3n^3-2} > \frac{1}{3n}$ for all $n \geq 1$, and $\sum_{n=1}^{\infty} \frac{1}{3n}$ diverges, we have by the Ordinary Comparison Test that $\sum_{n=1}^{\infty} \frac{n^2}{3n^3-2}$ also diverges.

2. Determine if the series $\sum_{n=1}^{\infty} \frac{n^{50}}{n!}$ converges or diverges.

   Solution: $\rho = \lim_{n\to\infty} \frac{(n+1)^{50}\, n!}{(n+1)!\, n^{50}} = \lim_{n\to\infty} \frac{1}{n+1}\left(\frac{n+1}{n}\right)^{50} = 0 < 1$ so by the Ratio Test, the series must converge.

3. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\ln(n+1)}$ converges or diverges.

   Solution: Note that the series is an alternating series. Since $\frac{1}{\ln(n+1)} > \frac{1}{\ln(n+2)}$ for all n and since $\lim_{n\to\infty} \frac{1}{\ln(n)} = 0$, the Alternating Series Test implies that the series converges.

4. Determine if the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt[3]{n}}$ is absolutely convergent, conditionally convergent, or divergent.

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Solution: Note that the series is an alternating series. Since $\frac{1}{\sqrt[3]{n}} > \frac{1}{\sqrt[3]{n+1}}$ for all n and since $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} = 0$, the Alternating Series Test implies the series converges. However, $\sum_{n=1}^{\infty} \left|(-1)^{n+1}\frac{1}{\sqrt[3]{n}}\right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ which diverges by the p-series Test. So the series is conditionally convergent.