Quiz 7 Solution - Math 166

Name: ______________________________

Show and justify **all** work to receive maximum credit for each problem. You may not use your book, notes, or a calculator on this quiz. Give exact answers, not decimal approximations. Do not give answers as mixed fractions. This quiz is worth 20 points.

1. (5 points) Evaluate $\lim_{n\to\infty} a_n$ where the sequence is defined by $a_n = \dfrac{2n^2}{3n^2 - 5n}$.

Solution: $\lim_{n\to\infty} a_n = \dfrac{2}{3}$

2. (3 points) Explain why the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2}{3n^2-5n}$ diverges.

Solution: Since $\displaystyle\lim_{n\to\infty} \frac{2n^2}{3n^2-5n} \neq 0$, it follows by the n^{th} Term Test that $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2}{3n^2-5n}$ diverges.

3. (6 points) Determine if the series $\displaystyle\sum_{k=1}^{\infty} 2\left(\frac{1}{3}\right)^{k-1}$ converges or diverges. If it converges, find the sum.

Solution: The series $\displaystyle\sum_{k=1}^{\infty} 2\left(\frac{1}{3}\right)^{k-1}$ converges to 3.

4. (6 points) Use the Integral Test to determine if the series $\sum_{k=1}^{\infty} ke^{-(k^2)}$ converges or diverges.

Solution: Note that $f(x) = xe^{-x^2}$ is continuous, positive, and nonincreasing on $[1,\infty)$. Also $\displaystyle\int_1^{\infty} xe^{-x^2}\,dx = \lim_{t\to\infty} -\frac{1}{2}e^{-t^2} + \frac{1}{2}e^{-1} = \frac{1}{2e}$, so by the Integral Test, $\sum_{k=1}^{\infty} ke^{-(k^2)}$ converges as well.