Exam 1 Solution - Math 166

Name: ______________________________

Do 5 of the following 6 problems. Each question is worth 20 points. Show all work to receive credit for each problem. If you turn in all 6 problems, your score will be based on the 5 highest scoring problems. You may not use your book, notes, or a calculator on this exam. Give exact answers, not decimal approximations. Do not give answers as mixed fractions.

1. (a) Find the area of the region bounded by the curves $y = 2x^2 - 4$ and $y = x^2 + 5$.

   Solution: $A = \int_{-3}^{3} (x^2 + 5) - (2x^2 - 4)\, dx = 36$

   (b) Find the area of the region in the first quadrant bounded by the curves $y = \dfrac{1}{2x+1}$, $y = 0$, $x = 0$, and $x = 7$.

   Solution: $A = \int_{0}^{7} \dfrac{1}{2x+1}\, dx = \dfrac{\ln(15)}{2}$

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2. Let R be the region in the first quadrant bounded by the curves $y = x^3$, $x = 0$, and $y = 8$.

   (a) Provide a sketch of the region R.

   (b) Suppose R is revolved about the axis $x = -1$. Find the volume of the resulting solid. You may use either method that you have learned.

Solution: $V = \pi \int_0^8 (y^{1/3} + 1)^2 - 1 \, dy = \pi \frac{216}{5}$

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3. Suppose a curve is given by the parametric equations $x(t) = 7\sin(2t)$, $y(t) = 7\cos(2t)$, for $0 \leq t \leq \frac{\pi}{2}$.

   (a) Show that this curve is smooth.

   Solution: $x'(t) = 14\cos(2t)$ and $y'(t) = -14\sin(2t)$, so both x and y are continuously differentiable. Since $x'(t)$ and $y'(t)$ are never both zero for the same t, we have by definition that this curve is smooth.

   (b) Using an integral, find the arc length of this curve.

   Solution: $L = \int_0^{\pi/2} \sqrt{14^2\cos^2(2t) + 14^2\sin^2(2t)}\, dt = 7\pi$

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4. Suppose you have a right circular cylindrical tank of height 6 meters and radius 2 meters. If the tank is exactly half-full of a liquid of density δ, use an integral to find the work necessary to pump this liquid out of the tank to a height 2 meters above the top of the tank.

Solution: $W = \int_0^3 4\delta\pi(8 - y)\, dy = 78\delta\pi$

5. A thin straight wire which is 5 meters long has density (kilograms per meter) at a point x units from one end given by $\delta(x) = 1 + x^3$.

   (a) Find the total mass of this wire.

   Solution: $m = \int_0^5 (1 + x^3)\, dx = 5 + \frac{625}{4}$

   (b) Find the distance from that end of the wire to the center of mass.

   Solution: $\bar{x} = \frac{\int_0^5 x(1 + x^3)\, dx}{m} = \frac{\frac{25}{2} + 5^4}{5 + \frac{625}{4}}$

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6. Use the techniques of this chapter to verify that the surface area of a sphere of radius R is $4\pi R^2$. Hint: the graph of the function $y = \sqrt{R^2 - x^2}$ for $-R \leq x \leq R$ is the upper half of the circle of radius R centered at the origin.

Solution: $A = \int_{-R}^{R} 2\pi\sqrt{R^2 - x^2}\sqrt{1 + \frac{x^2}{R^2 - x^2}}\, dx = 4\pi R^2$