

Quiz 3 Solution - Math 165

Name: _____

Show and justify all work to receive maximum credit for each problem. You may not use your book, notes, or a calculator on this quiz. Give exact answers, not decimal approximations. Do not give answers as mixed fractions. This quiz is worth 20 points.

1. (5 points) The function $f(x) = \frac{x^2 + x - 2}{x^2 - 5x + 4}$ is not defined for $x = 1$. How should $f(1)$ be defined so that f is continuous at 1?

Solution: $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x + 2}{x - 4} = -1$. So by defining $f(1) = -1$, we have that f is continuous at 1 by definition.

2. (5 points) Determine all points at which the function g given below is discontinuous. Determine whether each is removable or nonremovable. Justify your answers.

$$g(x) = \begin{cases} -5 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ x - 5 & \text{if } 0 < x < 5 \\ 2 & \text{if } x \geq 5 \end{cases}$$

Solution: g has a removable discontinuity at $x = 0$, which can be removed by redefining $g(0) = -5$, and g has a nonremovable discontinuity at $x = 5$.

3. (5 points) Use the Intermediate Value Theorem to show that $x^5 - 5\sqrt{x} + 2 = 0$ has a solution in the interval $[0, 1]$. (Do not find the value of the solution.)

Solution: Let $f(x) = x^5 - 5\sqrt{x} + 2$ which is continuous on $[0, 1]$. Note that $f(0) = 2$ and $f(1) = -2$, so by the Intermediate Value Theorem, there exists a c between 0 and 1 such that $f(c) = 0$, which satisfies the equation above.

4. (5 points) An object moves along a coordinate axis with position at time t given by $s(t) = \frac{7}{t+1}$. Use the limit method from Section 2.1 to find the object's instantaneous velocity at time $t = 2$.

Quiz 3 Solution - Math 165

$$\text{Solution: } v(2) = \lim_{h \rightarrow 0} \frac{s(2+h) - s(2)}{h} = \lim_{h \rightarrow 0} \frac{\frac{7}{3+h} - \frac{7}{3}}{h} = \lim_{h \rightarrow 0} \frac{21 - 21 - 7h}{3h(3+h)} = -\frac{7}{9}$$