

Math 165 - Homework Assignment 5 Solution

Name: _____

Write your solutions to these problems on a separate sheet of paper. Show **all** work to receive full credit for each problem. Give exact answers, not decimal approximations. This assignment is worth 10 points and is due **Monday, April 14** in class.

1. Determine if the following statements are true or false. If true, provide a short argument to justify your assertion. If false, provide a counterexample that demonstrates the statement is false.

(a) If $f(x) \geq 0$ for all x in $[a, b]$, then $\int_a^b f(x) dx \geq 0$.

Solution: True - use the Comparison Property

(b) If $\int_a^b f(x) dx \geq 0$, then $f(x) \geq 0$ for all x in $[a, b]$.

Solution: False - one such counterexample would be $f(x) = \sin(x)$ on $\left[0, \frac{3\pi}{2}\right]$

2. Evaluate the following definite integrals.

(a) $\int_0^2 x^2 \sqrt{x^3 + 1} dx$

Solution: $\int_0^2 x^2 \sqrt{x^3 + 1} dx = \frac{52}{9}$

(b) $\int_{-\sqrt{\pi}}^0 t \sin(t^2) \cos(t^2) dt$

Solution: $\int_{-\sqrt{\pi}}^0 t \sin(t^2) \cos(t^2) dt = 0$

(c) $\int_{-1}^4 [[x]] dx$

Solution: $\int_{-1}^4 [[x]] dx = 5$

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(d) $\int_{-10}^{10} \frac{x^3}{x^8 + 1} dx$ [Hint: Don't try looking for an antiderivative.]

Solution: $\int_{-10}^{10} \frac{x^3}{x^8 + 1} dx = 0$ as $f(x)$ is odd

3. Let $F(x) = \int_1^{x^2} t^2 dt$. Find the equation of the tangent line to the curve $y = F(x)$ at the point when $x = 2$.

Solution: $y - 21 = 64(x - 2)$