Math 165 - Homework Assignment 3 Solution

Name: ____________________

Write your solutions to these problems on a **separate** sheet of paper. Show **all** work to receive full credit for each problem. Turn in complete, legible, organized, and logically sound solutions and arguments. Give exact answers, not decimal approximations. This assignment is worth 10 points and is due **Tuesday, February 26** in class. I will grade all 3 problems.

1. Find $\dfrac{ds}{du}$ if $\tan(su^2) - \sqrt[3]{su+1} = \pi u$

   Solution: $$\frac{ds}{du} = \frac{\pi - 2su\sec^2(su^2) + \dfrac{s}{2(su+1)^{2/3}}}{u^2\sec^2(su^2) - \dfrac{u}{3(su+1)^{2/3}}}$$

2. You are climbing a rigid 20-foot ladder that is leaning against a tall vertical wall when the ladder begins to slide down the side of the wall. Assuming that the angle between the ground and the ladder is decreasing at a consant rate of $\frac{\pi}{6}$ radians per second, how fast is the top of the ladder moving down the side of the wall when the top of the ladder is 10 feet above the ground?

   Solution: Referring the picture, we are given that $\frac{d\theta}{dt} = -\frac{\pi}{6}$ and we are trying to find $\frac{dh}{dt}$. But $\sin(\theta) = \frac{h}{20} \Rightarrow \cos(\theta)\frac{d\theta}{dt} = \frac{1}{20}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = 20\cos(\theta)\frac{d\theta}{dt}$

   When $h = 10$, $\theta = \frac{\pi}{6} \Rightarrow \frac{dh}{dt} = 20\cos\left(\frac{\pi}{6}\right)\left(-\frac{\pi}{6}\right) = -\frac{5\pi\sqrt{3}}{3}$ feet per second

3. The water in a full conical metal tank (with vertex pointing upward!) begins to drain out at a rate of 4 cubic feet per minute. If the height of the tank is 18 feet and the diameter of the bottom of the tank is 12 feet, determine how fast the water level is dropping when the water is at a height of 10 feet.

   Solution: Referring to the picture, we are given $\frac{dV}{dt} = -4$ and we are trying to find $\frac{dh}{dt}$. The volume of water is given by $V = \frac{1}{3}\pi(6)^2(18) - \frac{1}{3}\pi r^2 h$. But due to similar right triangles, $\frac{r}{h} = \frac{6}{18} \Rightarrow r = \frac{h}{3}$.

   Thus $V = 36(6)\pi - \frac{\pi}{27}h^3 \Rightarrow \frac{dV}{dt} = -\frac{\pi h^2}{9}\frac{dh}{dt}$.

   When the water level is 10 feet, $h = 18 - 10 = 8 \Rightarrow \frac{dh}{dt} = \frac{9}{16}\pi$ feet per minute. Hence the water level is dropping at a rate of $-\frac{9}{16}\pi$ feet per minute.