Math 165 - Homework Assignment 2 Solution

Name: _____

Write your solutions to these problems on a **separate** sheet of paper. Show **all** work to receive full credit for each problem. Turn in complete, legible, organized, and logically sound solutions and arguments. Give exact answers, not decimal approximations. This assignment is worth 10 points and is due **Tuesday, February 5** in class. I will grade all 3 problems.

1. Evaluate the following limits. Be sure to state if a limit is of an infinite type.

   (a) $\lim_{x\to\infty} \cos\left(\frac{1}{x^2+1}\right)$

   Solution: $\lim_{x\to\infty} \cos\left(\frac{1}{x^2+1}\right) = \cos(0) = 1$

   (b) $\lim_{x\to 2} \frac{\sin(x^2-4)}{3x^2-12}$

   Solution: $\lim_{x\to 2} \frac{\sin(x^2-4)}{3x^2-12} = \frac{1}{3}\lim_{x\to 2} \frac{\sin(x^2-4)}{x^2-4} = \frac{1}{3}$

   (c) $\lim_{x\to c^-} \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}$

   Solution: $\lim_{x\to c^-} \frac{1}{\sqrt{1-\frac{x^2}{c^2}}} = \infty$

2. Suppose that f is continuous on $[0,1]$ such that $0 \le f(x) \le 1$ for all $0 \le x \le 1$. Show that f has a fixed point in $[0,1]$; that is, there is a number c in $[0,1]$ such that $f(c) = c$. [Hint: Consider the function $G(x) = x - f(x)$ and apply the Intermediate Value Theorem].

   Solution: Let $G(x) = x - f(x)$, which is continuous. So $G(0) = -f(0) \le 0$ and $G(1) = 1 - f(1) \ge 0$. So either $f(0) = 0$ and we have that 0 is a fixed point, or $f(1) = 1$ and we have that 1 is a fixed point, or else $G(0) < 0$ and $G(1) > 0$. In the latter case, the Intermediate Value Theorem implies the existence of a number $0 < c < 1$ such that $G(c) = 0 \Rightarrow c = f(c)$ and hence c is a fixed point.

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3. Find real numbers a, b, c, d such that the function H given below is continuous on $(-\infty, \infty)$. Justify your answers.

$$H(x) = \begin{cases} -3 & \text{if } x \leq -2 \\ ax + b & \text{if } -2 < x < 0 \\ 2 & \text{if } 0 \leq x \leq 3 \\ cx + d & \text{if } 3 < x < 4 \\ -1 & \text{if } x \geq 4 \end{cases}$$

Solution: The values that work are $a = \frac{5}{2}, b = 2, c = -3, d = 11$.