

Exam 4 Solution - Math 165

Name: _____

Show and justify all work to receive full credit for each problem. You may not use your book, notes, or a calculator on this exam. Give exact answers, not decimal approximations. Do not give answers as mixed fractions. This exam is worth 100 points.

1. (8 points) Evaluate $\int \frac{x-1}{x^2-2x+7} dx$

$$\text{Solution: } \int \frac{x-1}{x^2-2x+7} dx = \frac{1}{2} \ln|x^2-2x+7| + C$$

2. (8 points) Evaluate $\int \frac{x^2}{(x^3-2)^3} dx$

$$\text{Solution: } \int \frac{x^2}{(x^3-2)^3} dx = -\frac{1}{6(x^3-2)^2} + C$$

3. (8 points) Evaluate $\frac{dy}{dx}$ if $y = \sin(x) \ln(x^3+10)$

$$\text{Solution: } \frac{dy}{dx} = \cos(x) \ln(x^3+10) + \frac{3x^2 \sin(x)}{x^3+10}$$

4. (8 points) Evaluate $\frac{d}{dx} (10^{\tan(x)})$

$$\text{Solution: } \frac{d}{dx} (10^{\tan(x)}) = \ln(10) \sec^2(x) 10^{\tan(x)}$$

5. (8 points) Evaluate $\int_0^{\pi/2} \cos(x) e^{\sin(x)} dx$

$$\text{Solution: } \int_0^{\pi/2} \cos(x) e^{\sin(x)} dx = e - 1$$

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6. (8 points) Evaluate $\int_0^{\pi^2} \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

Solution: $\int_0^{\pi^2} \frac{\cos(\sqrt{x})}{\sqrt{x}} dx = 0$

7. (8 points) Evaluate $\frac{dy}{dx}$ if $y = \int_2^{x^2} \cos(t^2 + \pi) dt$

Solution: $\frac{dy}{dx} = 2x \cos(x^4 + \pi)$

8. (12 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = x^{(x^2)}$

Solution: $\frac{dy}{dx} = x^{(x^2)}(2x \ln(x) + x)$

9. (12 points) Let $g(x) = xe^{-x}$ on $(-\infty, \infty)$. Find all critical points for $g(x)$ and determine whether each is a local maximum or a local minimum.

Solution: The point where $x = 1$ is a local maximum.

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10. (12 points) Show that the function $f(x) = \cos(x)$ defined on the interval $(0, \pi)$ has an inverse. Then use the Inverse Function Theorem to find the value of $(f^{-1})'(\frac{1}{2})$.

Solution: $f'(x) = -\sin(x) < 0$ on $(0, \pi) \Rightarrow f$ is strictly decreasing and hence has an inverse, and $(f^{-1})'(\frac{1}{2}) = -\frac{2}{\sqrt{3}}$

11. (12 points) Use the **definition** of the Riemann integral to calculate the value of

$$\int_2^3 x^2 - 1 \, dx$$

Solution: $\int_2^3 x^2 - 1 \, dx = \frac{16}{3}$