Exam 1 Solution - Math 165

Name: ____________________

Show and justify all work to receive full credit for each problem. You may not use your book, notes, or a calculator on this quiz. Give exact answers, not decimal approximations. Do not give answers as mixed fractions. This exam is worth 100 points.

Be sure to indicate whether a given limit is of an infinite type or if a limit does not exist.

1. Evaluate $\lim_{x\to -3} \frac{x^2+x-6}{x^2+8x+15}$

Solution: $\lim_{x\to -3} \frac{x^2+x-6}{x^2+8x+15} = \lim_{x\to -3} \frac{x-2}{x+5} = -\frac{5}{2}$

2. Evaluate $\lim_{x\to \infty} \frac{3x^2-2x^3-8}{6x^3+x-1}$

Solution: $\lim_{x\to \infty} \frac{3x^2-2x^3-8}{6x^3+x-1} = -\frac{1}{3}$

3. Evaluate $\lim_{t\to 0} \frac{\tan(3t)}{\sin(2t)}$

Solution: $\lim_{t\to 0} \frac{\tan(3t)}{\sin(2t)} = \lim_{t\to 0} \frac{3(2t)\sin(3t)}{2(3t)\cos(3t)\sin(2t)} = \frac{3}{2}$

4. Evaluate $\lim_{t\to \pi} \frac{\sec(t)}{t^3}$

Solution: $\lim_{t\to \pi} \frac{\sec(t)}{t^3} = \frac{\sec(\pi)}{\pi^3} = -\frac{1}{\pi^3}$

5. Evaluate $\lim_{x\to 1^-} \frac{x+4}{x-1}$

Solution: $\lim_{x\to 1^-} \frac{x+4}{x-1} = -\infty$

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6. Evaluate $\frac{d}{dx}\left(\frac{x^2-1}{5x+2}\right)$

> **Solution:** $\frac{d}{dx}\left(\frac{x^2-1}{5x+2}\right) = \frac{(2x)(5x+2)-5(x^2-1)}{(5x+2)^2}$

7. Evaluate $\frac{d}{dx}\left(5x^4 - \frac{3}{2}x^2 + \pi\right)$

> **Solution:** $\frac{d}{dx}\left(5x^4 - \frac{3}{2}x^2 + \pi\right) = 20x^3 - 3x$

8. Show that $\lim_{x\to 6} \frac{|x-6|}{x-6}$ does not exist.

> **Solution:** $\lim_{x\to 6^+} \frac{|x-6|}{x-6} = \lim_{x\to 6^+} \frac{x-6}{x-6} = 1$ and $\lim_{x\to 6^-} \frac{|x-6|}{x-6} = \lim_{x\to 6^-} \frac{-(x-6)}{x-6} = -1$ so the limit does not exist.

9. Find the equation of the line tangent to the curve $y = \frac{1}{x^2}$ at $x = 2$.

> **Solution:** The equation of the tangent line is $y = -\frac{1}{4}x + \frac{3}{4}$

10. Use the definition of the derivative to find $f'(x)$ if $f(x) = \sqrt{x-3}$

> **Solution:** $f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h-3} - \sqrt{x-3}}{h} = \frac{1}{2\sqrt{x-3}}$

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11. Show that the equation $5x^5 - 2x^2 + x + 1 = 0$ has a solution in the interval $[-1, 1]$.

Solution: Let $f(x) = 5x^5 - 2x^2 + x + 1$ which is continuous as it is a polynomial. Note that $f(-1) = -7$ and $f(1) = 5$, so by the Intermediate Value Theorem, there exists a number $-1 < c < 1$ such that $f(c) = 0$ which means that $5c^5 - 2c^2 + c + 1 = 0$.

12. Use the Squeeze Theorem to show that $\lim_{x\to\infty} \frac{1}{x^2}\cos(x) = 0$ [Hint: Recall that it holds that $-1 \leq \cos(x) \leq 1$ for all x.]

Solution: $-1 \leq \cos(x) \leq 1 \Rightarrow \frac{-1}{x^2} \leq \frac{1}{x^2}\cos(x) \leq \frac{1}{x^2}$. Since $\lim_{x\to\infty} \frac{1}{x^2} = \lim_{x\to\infty} \frac{-1}{x^2} = 0$, the Squeeze Theorem implies that $\lim_{x\to\infty} \frac{1}{x^2}\cos(x) = 0$.

13. Determine all points of discontinuity for the function $g(x)$ given below. Determine whether each is removable or nonremovable. Justify your answers.

$$g(x) = \begin{cases} -x - 2 & \text{if } x < -2 \\ 1 & \text{if } x = -2 \\ 0 & \text{if } -2 < x < 0 \\ \cos(x) & \text{if } x \geq 0 \end{cases}$$

Solution: $x = -2$ is a removable discontinuity and $x = 0$ is a nonremovable discontinuity.