

## Worksheet 7 Solution - Math 165

Name: \_\_\_\_\_

Show all work to receive credit for each problem. All work must be *organized* and *legible*. Give exact answers, not decimal approximations. This assignment is worth 15 points and is due **Tuesday, April 24** in class.

1. Use logarithmic differentiation to find  $\frac{dy}{dx}$  if  $y = \frac{(x^3 + 1)^{4/3}}{\sqrt{2x^2 + 3}}$

$$\text{Solution: } \frac{dy}{dx} = \frac{(x^3 + 1)^{4/3}}{\sqrt{2x^2 + 3}} \left( \frac{4x^2}{x^3 + 1} - \frac{2x}{2x^2 + 3} \right)$$

2. Find all local extrema of the function  $f(x) = xe^{-2x^2}$  and classify each as a local maximum or a local minimum using the Second Derivative Test.

**Solution:**  $f'(x) = e^{-2x^2} - 4x^2e^{-2x^2}$ ,  $f''(x) = -4xe^{-2x^2} - 8xe^{-2x^2} + 16x^3e^{-2x^2}$  and  $f'(x) = 0 \Rightarrow x = \pm \frac{1}{2} \Rightarrow f''(1/2) < 0$ ,  $f''(-1/2) > 0$ , so  $f(1/2)$  is a local max and  $f(-1/2)$  is a local min.

3. Prove that the function  $f(x) = 5x^3 - 1$  is one-to-one. Then find its inverse.

**Solution:**  $f'(x) = 15x^2 > 0$  for all  $x \neq 0 \Rightarrow f$  is strictly monotone and hence one-to-one.  $f^{-1}(x) = \sqrt[3]{\frac{x+1}{5}}$

4. Evaluate the integral

$$\int_0^1 10^{2x} + 10^{-2x} dx$$

**Solution:**  $\int_0^1 10^{2x} + 10^{-2x} dx = \frac{50}{\ln(10)} - \frac{1}{200 \ln(10)}$

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5. Let  $g(x) = \log_2(7x)$ . Find the equation of the tangent line to the curve  $y = g(x)$  at the point when  $x = \frac{8}{7}$ .

**Solution:**  $y = \frac{7}{8 \ln(2)}x + \frac{20}{7}$