Worksheet 6 - Math 165

Name: _____

Show all work to receive credit for each problem. All work must be *organized* and *legible*. Give exact answers, not decimal approximations. This assignment is worth 15 points and is due **Tuesday, April 10** in class.

1. Evaluate
$$\int_1^7 \frac{1}{\sqrt{2x+2}}\,dx$$

Solution: $= 2$

2. Evaluate
$$\int_0^{\pi/6} \frac{\sin(t)}{\cos^5(t)}\,dt$$

Solution: $= \dfrac{7}{36}$

3. Find all inflection points of the function $F(x) = \int_0^x t^{8/3} dt$

Solution: $F'(x) = x^{8/3} \Rightarrow F''(x) = \dfrac{8}{3}x^{5/3} = 0$ when $x = 0$. F is concave up if $x > 0$ and concave down if $x < 0$ so $x = 0$ defines an inflection point of F.

4. Determine whether each of the following statements is true or false. If the statement is true, prove it. If it is false, provide a counterexample.
   (a) If $\int_a^b f(x)\,dx < 0$ then $f(x) \leq 0$ for all x in $[a, b]$.

   Solution: False. Let $f(x) = -1$ for $0 \leq x \leq \dfrac{3}{4}$ and $f(x) = 1$ for $\dfrac{3}{4} < x \leq 1$ $\Rightarrow \int_0^1 f(x)dx < 0$ but $f(x) > 0$ for some x in $[a, b]$.

   (b) Suppose f is continuous on $[a, b]$. Then $G(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$.

   Solution: True. $G'(x) = f(x)$, so G is differentiable on $[a, b] \Rightarrow G$ is continuous on $[a, b]$

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5. Let f be continous on $[a, b]$. Prove that $\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$.
(Hint: $-|f(x)| \leq f(x) \leq |f(x)|$ for all x)

Solution: $-|f(x)| \leq f(x) \leq |f(x)|$ for all x
$\Rightarrow -\int_a^b |f(x)|dx \leq \int_a^b f(x)dx \leq \int_a^b |f(x)|dx$
$\Rightarrow \left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$