

Worksheet 3 Solution - Math 165

Name: _____

Show all work to receive credit for each problem. All work must be *organized* and *legible*. Give exact answers, not decimal approximations. This assignment is worth 10 points and is due **Monday, February 19** in class.

1. Approximate $\sqrt[3]{126}$ using differentials (or equivalently, the linear approximation). Is your answer an overestimate or an underestimate? Why?

Solution: Let $f(x) = \sqrt[3]{x} \Rightarrow f'(x) = \frac{1}{3(\sqrt[3]{x})^2} \Rightarrow f(125+1) \approx f(125) + f'(125)(1) = 5 + \frac{1}{75} = \frac{376}{75}$. Comparing the function $f(x) = \sqrt[3]{x}$ to its tangent line at $x = 125$, our approximation is obviously an overestimate.

2. A spherical balloon is being inflated with air. Use differentials to approximate the increase in its surface area if the radius changes from 1 ft to 1.01 ft.

Solution: Let $f(x) = 4\pi x^2 \Rightarrow f'(x) = 8\pi x \Rightarrow f(1+0.01) - f(1) \approx f'(1)(0.01) = \frac{8\pi}{100}$

3. A metal rod has the shape of a right circular cylinder. As it is being heated, its length is increasing at a rate of $\frac{1}{200}$ cm/min and its diameter is increasing at a rate of $\frac{1}{500}$ cm/min. At what rate is the volume of the rod changing when the rod has length 40 cm and diameter 3 cm?

Solution: $V = \pi r^2 h \Rightarrow \frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt} \Rightarrow \frac{dV}{dt} = (2\pi)(1.5)(40)(1/1000) + (\pi)(1.5^2)(1/200) = \frac{3\pi}{25} + \frac{9\pi}{800}$ cm³/min.

4. The area of an equilateral triangle is decreasing at a rate of 4 cm²/s. Find the rate at which the length of a side is changing when the area of the triangle is 200 cm².

Solution: Let s denote the length of a side of the triangle. $A = \frac{\sqrt{3}s^2}{4}$ using the Pythagorean Theorem $\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}s \frac{ds}{dt} \Rightarrow \frac{ds}{dt} = \frac{400}{(\sqrt{800}/\sqrt{3})\sqrt{3}}$