Worksheet 1 Solutions - Math 165

Name: ______________________________

Show all work to receive credit for each problem. All work must be *organized* and *legible*. Give exact answers, not decimal approximations. This assignment is worth 10 points and is due **Tuesday, January 30** in class.

1. Let $f(x) = \dfrac{x^2 + 4x + 3}{2(x^2 - 1)}$.

   (a) Find all points of discontinuity of $f(x)$

   Solution: There are discontinuities at $x = 1$ and $x = -1$

   (b) Evaluate $\lim\limits_{x \to -1} f(x)$

   Solution: $\lim\limits_{x \to -1} f(x) = -\dfrac{1}{2}$

   (c) Evaluate $\lim\limits_{x \to \infty} f(x)$

   Solution: $\lim\limits_{x \to \infty} f(x) = \dfrac{1}{2}$

2. Let $g(x) = \sqrt{2x + 5}$. Use the definition of the derivative to find $g'(x)$.

   Solution: $g'(x) = \dfrac{1}{\sqrt{2x + 5}}$

3. Find the vertical and horizontal asymptotes of the function $h(x) = \dfrac{5x^2 + 1}{x^2 - 7x + 10}$

   Solution: $y = 5$ is the horizontal asymptote, $x = 5$ and $x = 2$ are vertical asymptotes.

4. Find $D_x \left[\dfrac{x^2}{10x^3 - x^2 + \pi} \right]$ – do not simplify your answer.

   Solution: $D_x \left[\dfrac{x^2}{10x^3 - x^2 + \pi} \right] = \dfrac{(2x)(10x^3 - x^2 + \pi) - (x^2)(30x^2 - 2x)}{(10x^3 - x^2 + \pi)^2}$

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5. An object moves along a line with position $s(t) = 5t^2 - 2t$ at time t seconds.

   (a) What is the object's average velocity between $t = 1$ and $t = 3$?

   Solution: $v_{avg} = 18$ units per second

   (b) What is the object's instantaneous velocity at time t?

   Solution: $v(t) = 10t - 2$

   (c) What is the object's position when its velocity is zero?

   Solution: $-\frac{1}{5}$ units

6. Using the Intermediate Value Theorem, prove that the equation $x^4 + 8x - 1 = 0$ has a solution between $x = 0$ and $x = 1$.

   Solution: Let $f(x) = x^4 + 8x - 1$, which is a continuous function. $f(0) = -1$ and $f(1) = 8$, so by the IVT there exists a c between 0 and 1 so that $f(c) = 0 \Rightarrow c^4 + 8c - 1 = 0$

7. Let $p(t) = \dfrac{\sin(4t)}{3t}$, $t \neq 0$. How should $p(0)$ be defined so that p is continuous everywhere?

   Solution: Define $p(0) = \dfrac{4}{3}$