

## Quiz 6 Solution - Math 165

Name: \_\_\_\_\_

Show all work to receive maximum credit for each problem. You may not use your book, notes, or a calculator on this quiz. Give exact answers, not decimal approximations. This quiz is worth 25 points.

1. (5 points) Let  $f(x) = \sqrt{x^2 + 1}$ . Find all critical points of  $f$  on the interval  $I = [-3, 1]$ .

**Solution:**  $f'(x) = \frac{x}{\sqrt{x^2 + 1}} \Rightarrow f$  has no singular points and a stationary point at  $x = 0$ . The critical points are  $x = -3, 0$ , and  $1$ .

2. (5 points) Let  $g(x) = \frac{1}{12}x^4 + x^3 - \frac{7}{2}x^2 + x - \frac{1}{2}$ . Find where  $g$  is concave up and where it is concave down.

**Solution:**  $g''(x) = x^2 + 6x - 7 = (x + 7)(x - 1) = 0$  when  $x = 1$  or  $-7$ , so  $g$  is concave up on  $(-\infty, -7)$  and  $(1, \infty)$ , and  $g$  is concave down on  $(-7, 1)$ .

3. (5 points) Use either the First or Second Derivative Test to find all local maxima and minima of the function  $h(x) = \frac{2}{3}x^3 - 8x + \frac{6}{5}$ , on  $(-\infty, \infty)$ .

**Solution:**  $h(-2)$  is a local maximum and  $h(2)$  is a local minimum.

4. (5 points) Show that  $f(x) = x^4$  has no inflection points.

**Solution:**  $f''(x) = 12x^2 \Rightarrow$  the only possible inflection point is at  $x = 0$ , but the function is concave up on all other points, so  $x = 0$  does not give an inflection point.

5. (5 points) Prove that a rectangle of fixed (constant) perimeter  $P$  with maximum area is a square.

**Solution:**  $A = lw = (\frac{P}{2} - w)(w) = \frac{P}{2}w - w^2 \Rightarrow A' = \frac{P}{2} - 2w = 0$  when  $w = \frac{P}{4} \Rightarrow l = \frac{P}{4}$  so it is a square.