

## Quiz 5 Solution - Math 165

Name: \_\_\_\_\_

Show all work to receive maximum credit for each problem. You may not use your book, notes, or a calculator on this quiz. Give exact answers, not decimal approximations. This quiz is worth 25 points.

1. (5 points) Find  $dy$  if  $y = \frac{\pi}{3}x^2 - x$ .

$$\text{Solution: } dy = \left( \frac{2\pi}{3}x - 1 \right) dx$$

2. (5 points) The volume  $V$  of an object is *decreasing* at a rate of  $5 \text{ in}^3/\text{sec}$ . Express this quantity in terms of a derivative, where  $t$  is time in seconds.

$$\text{Solution: } \frac{dV}{dt} = -5$$

3. (5 points) A circular metal disk is being heated so that the area  $A$  of one of its sides is increasing at a rate of  $2 \text{ mm}^2/\text{s}$ . How fast is the radius  $r$  increasing when the radius is  $500 \text{ mm}$ ?

$$\text{Solution: } A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow 2 = 2\pi(500) \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{500\pi} \text{ mm/sec}$$

4. (5 points) Find the linear approximation  $L(x)$  to the function  $f(x) = x + \cos^2(x)$  at the point  $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ .

$$\text{Solution: } f'(x) = 1 + 2\cos(x)\sin(x) \Rightarrow L(x) = f(\pi/2) + f'(\pi/2)(x - \pi/2) \\ \Rightarrow L(x) = \frac{\pi}{2} + x - \frac{\pi}{2} \Rightarrow L(x) = x$$

5. (5 points) Use differentials (or equivalently, the linear approximation) to approximate  $\sqrt{37}$ . [Hint: Let  $f(x) = \sqrt{x}$ , and  $37 = 36 + 1$ ]

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**Solution:** Let  $f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f(36 + 1) \approx f(36) + f'(36)(1) =$   
 $6 + \frac{1}{12} = \frac{73}{12} \Rightarrow \sqrt{37} \approx \frac{73}{12}$