

### Quiz 3 Solution - Math 165

Name: \_\_\_\_\_

Show all work to receive maximum credit for each problem. You may not use your book, notes, or a calculator on this quiz. Give exact answers, not decimal approximations. This quiz is worth 25 points.

1. (6 points) Let  $g(x) = \frac{x^2 + x - 12}{x - 3}$  for  $x \neq 3$ . How should  $g(3)$  be defined so that  $g$  is continuous everywhere?

$$\text{Solution: } \lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3} = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 4)}{x - 3} = \lim_{x \rightarrow 3} x + 4 = 7, \text{ so define } g(3) = 7$$

2. (6 points) Suppose an object is moving so that its position at time  $t$  seconds is given by the function  $s(t) = \frac{1}{2}t^2 - 4t$ . What is the object's average velocity between times  $t = 0$  and  $t = 4$ ?

$$\text{Solution: } v_{avg} = \frac{s(4) - s(0)}{4 - 0} = \frac{-8 - 0}{4} = -2$$

3. (6 points) Use the **Intermediate Value Theorem** to show that the equation  $x^3 - 7x + 1 = 0$  has a solution between  $x = 0$  and  $x = 1$ .

**Solution:** Note that  $f(x) = x^3 - 7x + 1$  is continuous.  $f(0) = 1$  and  $f(1) = -5$ , so there exists  $c$  between 0 and 1 such that  $f(c) = 0 \Rightarrow c^3 - 7c + 1 = 0$

4. (7 points) Let  $f(x) = x^2 + 1$ . Use the definition of the derivative to show that  $f'(x) = 2x$ .

$$\begin{aligned} \text{Solution: } f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 1 - (x^2 + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} 2x + h = 2x \end{aligned}$$