

Quiz 2 Solution - Math 165

Name: _____

Show all work to receive maximum credit for each problem. You may not use your book, notes, or a calculator on this quiz. Give exact answers, not decimal approximations. This quiz is worth 25 points.

1. (5 points) Evaluate $\lim_{x \rightarrow 0} \frac{\sin(5x)}{x}$

$$\text{Solution: } \lim_{x \rightarrow 0} \frac{\sin(5x)}{x} = \lim_{x \rightarrow 0} \frac{5 \sin(5x)}{5x} = 5 \lim_{x \rightarrow 0} \frac{\sin(5x)}{5x} = 5$$

2. (5 points) Evaluate $\lim_{t \rightarrow 0} \frac{\tan(t)}{3t}$

$$\text{Solution: } \lim_{t \rightarrow 0} \frac{\tan(t)}{3t} = \lim_{t \rightarrow 0} \frac{\sin(t)}{3t \cos(t)} = \lim_{t \rightarrow 0} \frac{\sin(t)}{t} * \lim_{t \rightarrow 0} \frac{1}{3 \cos(t)} = 1 * \frac{1}{3} = \frac{1}{3}$$

3. (5 points) Evaluate $\lim_{x \rightarrow \infty} \frac{x^2}{x^2 + 2}$

$$\text{Solution: } \lim_{x \rightarrow \infty} \frac{x^2}{x^2 + 2} = \lim_{x \rightarrow \infty} \frac{x^2/x^2}{x^2/x^2 + 2/x^2} = \lim_{x \rightarrow \infty} \frac{1}{1 + 2/x^2} = \frac{1}{1 + 0} = 1$$

4. (5 points) Evaluate $\lim_{x \rightarrow \infty} \frac{4x - 7}{3x + 1}$

$$\text{Solution: } \lim_{x \rightarrow \infty} \frac{4x - 7}{3x + 1} = \lim_{x \rightarrow \infty} \frac{4x/x - 7/x}{3x/x + 1/x} = \lim_{x \rightarrow \infty} \frac{4 - 7/x}{3 + 1/x} = \frac{4 - 0}{3 + 0} = \frac{4}{3}$$

5. (5 points) Prove that $\lim_{t \rightarrow c} \cot(t) = \cot(c)$ for all c such that $\cot(c)$ is defined. [You may assume that we know $\lim_{t \rightarrow c} \sin(t) = \sin(c)$, and $\lim_{t \rightarrow c} \cos(t) = \cos(c)$. Show all steps.]

$$\text{Solution: } \lim_{t \rightarrow c} \cot(t) = \lim_{t \rightarrow c} \frac{\cos(t)}{\sin(t)} = \frac{\lim_{t \rightarrow c} \cos(t)}{\lim_{t \rightarrow c} \sin(t)} = \frac{\cos(c)}{\sin(c)} = \cot(c)$$