

Exam 1 Solutions - Math 165

Name: _____

You may not use your book or notes on this exam, but you may use a calculator. **All work must be shown to receive credit.** Give exact answers, not decimal approximations. This exam is worth 100 points. If you need extra room to work a problem, use the back of the sheet.

1. (5 points) Evaluate $\lim_{x \rightarrow -1} \frac{x^2 + 4x + 3}{x^2 - 4x - 5}$

Solution: $= -\frac{1}{3}$

2. (5 points) Evaluate $\lim_{t \rightarrow 0} \frac{\tan(4t)}{7t}$

Solution: $= \frac{4}{7}$

3. (5 points) Evaluate $\lim_{x \rightarrow -\infty} \frac{6x^3 - 7x + 1}{3x^3 + 2x^2 - 9}$

Solution: $= 2$

4. (5 points) Evaluate $\lim_{t \rightarrow 7} 2\pi$

Solution: $= 2\pi$

5. (5 points) Find $\frac{dy}{dx}$ if $y = \frac{1}{3}x^2 - 10x + \frac{1}{15}$

Solution: $\frac{dy}{dx} = \frac{2}{3}x - 10$

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6. (5 points) Find $f'(t)$ if $f(t) = 3t \sin(t)$

$$\text{Solution: } f'(t) = 3 \sin(t) + 3t \cos(t)$$

7. (5 points) Evaluate $D_x \left[\frac{x^2 + 1}{6x - 5} \right]$

$$\text{Solution: } = \frac{(2x)(6x - 5) - (6)(x^2 + 1)}{(6x - 5)^2}$$

8. (5 points) Evaluate $D_x[\tan(x) - 7x]$

$$\text{Solution: } = \sec^2(x) - 7$$

9. (10 points) An object moves along a line with position $s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 6t + 1$ at time t seconds. At what times is the instantaneous velocity of the object equal to zero?

$$\text{Solution: } s'(t) = t^2 - 7t + 6 = (t - 6)(t - 1) = 0 \Rightarrow t = 1 \text{ or } t = 6$$

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10. (10 points) What is the equation of the tangent line to the curve $y = 2 \cos(x) - \sin(x)$ at the point when $x = \frac{\pi}{2}$?

Solution: $y' = -2 \sin(x) - \cos(x) \Rightarrow m = -2 \Rightarrow y + 1 = -2 \left(x - \frac{\pi}{2} \right)$

11. (10 points) Assume f and g are both continuous at the point c . Show that $f - g$ is also continuous at c . Show all detailed steps.

Solution: $\lim_{x \rightarrow c} f(x) - g(x) = \lim_{x \rightarrow c} f(x) - \lim_{x \rightarrow c} g(x) = f(c) - g(c)$

12. (10 points) Let $h(x) = \frac{1}{x-1}$. Does the Intermediate Value Theorem imply the existence of a number c between -2 and 2 such that $h(c) = 0$? Does such a number exist? Explain your answers.

Solution: No, the function is not continuous on $[-2, 2]$ so the IVT does not apply. There exists no x such that $\frac{1}{x-1} = 0$. Note that $y = 0$ is a horizontal asymptote for h .

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13. (10 points) Define

$$f(x) = \begin{cases} \frac{(x+1)(x-2)(x+3)}{(x-2)(x+3)} & \text{if } x \neq 2, x \neq -3 \\ 0 & \text{if } x = 2 \\ -2 & \text{if } x = -3 \end{cases}$$

Is f a continuous function? Why? If not, find all points of discontinuity.

Solution: f is not continuous. There is a point of discontinuity at $x = 2$, as $\lim_{x \rightarrow 2} f(x) \neq f(2)$.

14. (10 points) Use the definition of the derivative to calculate $f'(x)$ if $f(x) = 3x^2 + x$.

Solution: $f'(x) = 6x + 1$